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The Properties of Cointegration Tests in Models with Structural Change

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Abstract

In this paper we examine, by means of Monte Carlo simulation, the properties of several cointegration tests when long run parameters are subject to structural changes. We allow for different types of stochastic and deterministic regime shifts, more specifically, changes governed by Markov chains, martingale parameter variation, sudden multiple breaks and gradual changes. Our Monte Carlo analysis reveals that tests with cointegration as the null hypothesis perform badly, while tests with the null of no cointegration retain much of their usefulness in this context.

Key Words: Structural change; Cointegration; Tests; Monte Carlo

JEL Classification: C12; C22; C52

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1 Introduction

The concept of cointegration has dominated the debate in time-series econometrics in the past decade, by stressing the possible existence of long-run equilibrium relationships among non-stationary variables. More recently, researchers became concerned with the effects that structural changes may have on econometric models. Indeed, failure to detect and account for parameter shifts is a serious form of misspecification, thus affecting inference and leading to poor forecasting performances (see Clements and Hendry, 1999). This is especially relevant for cointegration analysis, since it normally involves long spans of data, which, consequently, are more likely to display structural breaks.

In this paper, we investigate, by means of Monte Carlo experiments, the impact of various forms of parameter changes in the finite-sample properties of several cointegration tests. Previous literature on structural change and cointegration has focused on developing procedures to detect breaks or to estimate the temporal location of eventual shifts. See Hansen (1992), Quintos and Phillips (1993), Hao (1996), Andrews, Ploberger and Lee (1996), Bai, Lumsdaine and Stock (1998), Seo (1998) and Kuo (1998), among others, and Maddala and Kim (1998) for a general survey.

While there is a vast literature on the effect of structural breaks on univariate time series (see Maddala and Kim, 1998 and Stock, 1994), specifically dealing with the effect of parameter non-constancy on cointegration tests we have the works of Gregory, Nason and Watt (1996), whose conclusions are supported by Gregory and Hansen (1996), and Campos, Ericsson and Hendry (1996). However, these studies are somewhat limited in scope, in the sense that they only address one type of structural break (single deterministic jump) and concentrate on the properties of the Augmented Engle-Granger (AEG) cointegration test.

Therefore, our paper extends these studies by analyzing the power properties of cointegration tests with the null hypothesis of no cointegration (namely AEG, Phillips-Ouliaris and Gregory-Hansen tests), as well as tests with the null of cointegration (L_c test of Hansen, 1992 and the Shin-Harris-Inder test). If the researcher is interested in using tests for both the null of cointegration and no cointegration for confirmatory analysis¹, then it would be important to understand how these tests are affected by parameter shifts. Furthermore, we consider cointegration models where parameters are subject to multiple deterministic breaks, gradual shifts, random walk

¹Despite the problems with this approach (see discussion in Maddala and Kim, 1998, p. 126-128).

variation and Markov regime switching.

While trying to investigate the possible existence of parameter non-constancy in multivariate models with non-stationary variables, one resorts to instability tests that, in general, will only be valid if there the variables are, in fact, cointegrated. To verify this, one has to test for cointegration, hence the importance of understanding the properties of different cointegration tests when parameter changes occur. Moreover, our analysis stresses parameter non-constancy that is empirically plausible and economically meaningful in this context (i.e. not excessively large breaks), since for big enough breaks the properties of the tests would probably be more straightforward to examine (see discussion in Hendry, 1999).

The paper proceeds as follows. The next section reviews the cointegration tests of interest. Section 3 describes the experimental design of our simulations. Section 4 reports and discusses the results of the experiments and Section 5 concludes.

2 Cointegration Tests

In this section, we provide a necessarily brief description of the cointegration tests examined in the subsequent Monte Carlo study. Given the model

$$y_{1t} = \alpha y_{2t} + u_t; \quad (1)$$

where $y_t = (y_{1t}, y_{2t})$ is a $k \times 1$ vector of $I(1)$ variables, y_{2t} possibly containing deterministic elements (such as a constant or a trend), the variables in y_t will be cointegrated with cointegration vector $(1; \alpha')$ if u_t is stationary. To test this hypothesis in this paper, we employ "standard" tests with the null hypothesis of no cointegration, tests with cointegration as the null, as well as tests allowing for regime shifts.

2.1 Standard Cointegration Tests

The AEG and the Z_α and Z_t tests of Phillips and Ouliaris (1990) are unquestionably the most popular cointegration tests, having been extensively discussed in the literature. They may be viewed as an application of their unit-root counterparts (Augmented Dickey-Fuller and Phillips-Perron tests) to test whether the residuals $e_t = y_{1t} - \alpha y_{2t}$ from (1) have a unit-root or, by contrast, are stationary. While the AEG test corrects for serial correlation by adding lagged e_t terms in the test regression $e_t = (\alpha - 1)e_{t-1} + \epsilon_t$, Phillips-Ouliaris tests make use of a

nonparametric modification, which involves the estimation of γ^2 , the long run variance of the second-stage errors ϵ_t .

To select an appropriate lag length for the AEG test, we follow a t-test downward selection procedure, by setting the maximum lag equal to 6 and then testing downward until a significant last lag is found, at the 5% level. Finite-sample critical values computed as in MacKinnon (1991) will be used in our experiments. Turning to the Z_α and Z_t tests, the long run variance γ^2 is estimated by means of a prewhitened quadratic spectral kernel with an automatically selected bandwidth estimator, using a first-order autoregression as a prewhitening filter, as recommended in Andrews and Monahan (1992).

2.2 Gregory-Hansen Tests

Gregory and Hansen (1996) generalized the standard cointegration tests by considering an alternative hypothesis in which the cointegration vector may suffer a regime shift at an unknown timing. They analyzed models that accommodate, under the alternative, the possibility of changes in parameters, as, for example, the "regime shift" model

$$y_t = \alpha_1 + \alpha_2 D_t + \beta_1 t + \gamma_1 x_t + \gamma_2 x_t D_t + u_t; \quad t = 1, \dots, T; \quad (2)$$

where x_t is a k -dimensional vector of $I(1)$ variables, u_t should be stationary and D_t is a dummy variable of the type

$$D_t = \begin{cases} 0; & \text{if } t > [T\lambda] \\ 1; & \text{if } t \leq [T\lambda] \end{cases} \quad (3)$$

Here, $\lambda \in [0, 1]$ denotes the unknown relative timing of the break point and $[\cdot]$ denotes the integer part. Several types of models may be considered, for instance, a deterministic trend may be included, or the shift affects only the intercept, or only the slope coefficient, and so on.

As the previous tests, these are residual-based cointegration tests that evaluate if the error term is $I(1)$ under the null. In this framework, however, since the change point or its occurrence are unknown, the testing procedures involve the computation of the usual statistics for all possible break points $\lambda \in [0, 1]$ and then selecting the smallest value obtained, since it will potentially present greater evidence against the null hypothesis of no cointegration. Therefore, one should observe the values of

$$Z_\alpha = \inf_{\lambda \in [0, 1]} Z_\alpha; \\ Z_t = \inf_{\lambda \in [0, 1]} Z_t;$$

$$AEG^a = \inf_{j \in J} AEG_j:$$

The trimming region defined by J may be any compact set of $(0; 1)$; but following earlier literature, Gregory and Hansen (1996) propose $J = (0.15; 0.85)$. Nevertheless, it should be pointed out that these tests possess power against other alternatives, namely, "stable" cointegration. Hence, a rejection of the null hypothesis does not necessarily imply changes in the cointegration vector, since an invariant relationship might be the cause of the rejection.

These test statistics possess non-standard limiting distributions with no closed form and, therefore, critical values were obtained resorting to simulation methods. In this paper, we examine a type of structural break that is not tabulated, which is the change in slope alone. For proper comparison, and following Gregory and Hansen (1996), we obtained critical values for this type of change, with a single regressor, using the same response surface: with 10 000 replications for sample sizes $T = 50; 100; 150; 200; 250$ and 300 , critical values at the p percent level are obtained and then the regression

$$C(p; T) = \tilde{A}_0 + \tilde{A}_1 T^{1/2} + \text{error},$$

is run. The critical values at the 1%, 5% and 10% significance levels are, respectively, $\hat{c}_{1\%} = 5.268$, $\hat{c}_{5\%} = 4.685$ and $\hat{c}_{10\%} = 4.394$ for the AEG^a and Z_t^a tests, and $\hat{c}_{1\%} = 4.9159$, $\hat{c}_{5\%} = 3.9172$ and $\hat{c}_{10\%} = 3.4011$ for the Z_{∞}^a test.

2.3 Tests with Cointegration as the Null Hypothesis

The tests described in the previous sections are based on the principle of testing for a unit root in the residuals of the cointegrating regression. Other tests have been developed which test the stationarity of the residuals and, therefore, have cointegration as the null hypothesis. Since we are interested in the effects of neglected parameter changes, it is interesting to consider cointegration tests that may be derived from structural change tests.

Hansen (1992) proposed some LM-type structural change tests in cointegrated models, making use of the Fully-Modified OLS method. A versatile feature of those tests is the possibility of using them as cointegration tests. In fact, if the alternative hypothesis is that the intercept follows a random walk, then structural change testing becomes cointegration testing, albeit with the null hypothesis of cointegration. Rewriting model (1) as $y_t = \alpha_1 + \alpha_2 x_{2t} + u_t$, if y_t and x_{2t} are not cointegrated, then the error term u_t is integrated of order one. Decomposing u_t such

that $u_t = w_t + v_t$; being w_t a random walk and v_t a stationary term, the model then becomes

$$y_t = \alpha_{1t} + \alpha_{2t} x_{2t} + v_t; \quad (4)$$

with $\alpha_{1t} = \alpha_1 + w_t$; that is, the intercept "absorbs" the random walk w_t when there is no cointegration.

Having this fact in consideration, Hansen (1992) suggested the use of the statistic

$$L_c = T^{-1} \sum_{t=1}^T \hat{S}_t' \hat{V}_t^{-1} \hat{S}_t;$$

to test the null of cointegration, where $\hat{S}_t$ represents the scores of the FM-OLS estimates and $\hat{V}_t^{-1}$ is a weighting matrix based upon an estimate of the covariance matrix of the second-order errors. However, this statistic was designed to test the stability of the whole cointegration vector, so there are advantages in regarding a version that tests only (partial) structural change in the intercept. Hao (1996) has shown that such a test may be carried out by employing a known statistic, already used by Kwiatkowski, Phillips, Schmidt and Shin (1992) to test for stationarity, as well as Shin (1994) and Harris and Inder (1994) to test for the null of cointegration. Here, we use the latter version based on the FM-OLS estimator,

$$S^+ = \frac{T^{-2} \sum_{t=1}^T \left(\sum_{i=1}^p \hat{u}_i^+ \right)^2}{\hat{\sigma}_{1:2}^2};$$

where $\hat{u}_i^+$ represents the fully-modified residuals from the cointegrating regression and $\hat{\sigma}_{1:2}^2$ an estimate of the long run variance of u_t conditional on Φx_t . As with the Phillips-Ouliaris tests, the same non-parametric procedure described earlier is used in the FM-OLS estimation.

It is important, however, to stress that a researcher should be cautious in interpreting these tests, since a rejection does not entangle the immediate acceptance of the alternative hypothesis for which they were constructed. For instance, a rejection by the Shin-Harris-Inder (SHI) test does not mean that there is no cointegration, since it also has power against parameter instability. The only plausible conclusion one can draw is that the traditional specification of a cointegration model such as (1) (assuming parameter stability) is not supported by the data. The same applies to structural change tests used as cointegration tests.

3 Monte Carlo Analysis

In this section, we use Monte Carlo methods to evaluate the finite-sample properties of the cointegration procedures discussed above, when we allow for cointegration with changes in parameters.

First, we describe the different data-generating processes (DGP) and the experimental design used in the simulations. This is followed, in the next section, by a discussion of the numerical results.

We base our experiments on a simple, general model,

$$\begin{aligned} y_t &= \alpha_t + \beta_t x_t + u_t; \\ x_t &= x_{t-1} + \varepsilon_t; \quad t = 1, \dots, T; \end{aligned} \quad (5)$$

where y_t and x_t are both scalar and $\varepsilon_t \sim \text{i.i.d.}(0, 1)$, uncorrelated with u_t . For every DGP, the error term u_t is generated as an autoregressive process $u_t = \rho u_{t-1} + \eta_t$; $\eta_t \sim \text{i.i.d.}(0, 1)$, with $\rho = 0$; $\rho = 0.5$ and $\rho = 1$ ². The idea is to evaluate the tests properties with different error structures, since in an applied work context the disturbances are likely to be, at least, serially correlated. The selected sample sizes are $T = 50; 100$; and 200 :

We study the performance of the tests with changes occurring in the intercept (α_t), or in the intercept and the slope coefficient (β_t). The shifts are generated by different types of mechanisms: random walk parameter variation, sudden and gradual deterministic breaks and Markov regime changes, as described next.

The most extreme case of parameter change is a model where parameters vary with each time period t . We stipulate that the parameter non-constancy takes the form of a random walk, with the slope evolving as

$$\beta_t = \beta_{t-1} + \gamma_t; \quad \gamma_t \sim \text{i.i.d.}(0, \frac{1}{4}) \quad (6)$$

and set $\frac{1}{4} = 0.2$ and $\frac{1}{4} = 0.05$, following Kuo (1998). We concentrate only on changes in the slope coefficient, since this type of instability in the intercept would just be equivalent to no cointegration.

In addition, we specify a model where the coefficients shift twice, with the jumps specified as

$$\alpha_t = \begin{cases} \alpha_0 & t \in [T_{\alpha 1}] \\ \alpha_0 + \phi_{\alpha 1} & [T_{\alpha 1}] < t \leq [T_{\alpha 2}] \\ (\alpha_0 + \phi_{\alpha 1}) + \phi_{\alpha 2} & t > [T_{\alpha 2}] \end{cases}; \quad \beta_t = \begin{cases} \beta_0 & t \in [T_{\beta 1}] \\ \beta_0 + \phi_{\beta 1} & [T_{\beta 1}] < t \leq [T_{\beta 2}] \\ (\beta_0 + \phi_{\beta 1}) + \phi_{\beta 2} & t > [T_{\beta 2}] \end{cases}$$

The possible combinations in this case are innumerable. The sizes of the first break are $\phi_{\alpha 1} = (1; 3)$ and $\phi_{\beta 1} = (0.5; 1)$, while the size of the second shift varies with $\phi_{\alpha 2} = (-3; -1; 1; 3)$ and

²The case $\rho = 1$ corresponds to no cointegration and is generated in order to compute empirical sizes for tests with no cointegration as the null hypothesis.

$\Phi_{-1} = (\gamma_1; \gamma_0; 0.5; 1)$, with initial values $\gamma_0 = \gamma_{-0} = 1$. For a given break point $\lambda_1 = 0.25$ or $\lambda_1 = 0.5$, the second break point λ_2 can take values $\lambda_2 = 0.5; 0.75$.

In many situations, changes are more likely to be gradual rather than sudden, due to adjustment costs or anticipation effects. Thus, it seems reasonable to include in this study a DGP where parameters change gradually. Regarding this, we use a transition function discussed in Lin and Teräsvirta (1994), for example, based upon the smooth transition function

$$F(t) = [1 + \exp(-\gamma(t - \lambda))^\alpha]^{-1};$$

where α is a "speed of adjustment" parameter, around the break point λ , implying an immediate jump as $\alpha \rightarrow 1$. The experiments were performed with $\alpha = 0.5$ and with a slower "speed of adjustment" $\alpha = 0.1$, with changes in parameters derived from

$$\gamma_t = \begin{cases} \gamma_0 \\ \gamma_0 + \Phi_1 F(t) \end{cases}; \quad \gamma_{-t} = \begin{cases} \gamma_{-0} \\ \gamma_{-0} + \Phi_{-1} F(t) \end{cases}; \quad t \leq [\lambda]; \quad t > [\lambda];$$

for $\lambda = 0.25; 0.5$ and 0.75 and break sizes $\Phi = (\Phi_1; \Phi_{-1}) \sim f(1; 3); (0.5; 1)g$, with $\Phi_1 = \gamma_1 - \gamma_0$ and $\Phi_{-1} = \gamma_{-1} - \gamma_{-0}$.

Finally, we consider Markov-switching cointegration, as in Hall, Psaradakis and Sola (1997), where long run parameters switch between different cointegrating regimes. We continue to assume that x_t is generated by a driftless random walk, whereas y_t is now given by

$$y_t = \gamma_1(s_t) + \gamma_{-1}(s_t)x_t + u_t;$$

with

$$\begin{aligned} \gamma_{1t} &= \gamma_1(s_t) = \gamma_0 + (\gamma_1 - \gamma_0)(s_t - 1); \\ \gamma_{-t} &= \gamma_{-1}(s_t) = \gamma_{-0} + (\gamma_{-1} - \gamma_{-0})(s_t - 1); \end{aligned}$$

where s_t is a binary random variable in $S = \{0; 1\}$, indicating the unobserved regime or state of the cointegrating relationship, at date t . It is postulated that $\{s_t\}$ is a stationary first-order Markov chain in S with transition matrix $P = (p_{ij})$, where

$$p_{ij} = \Pr(s_t = j | s_{t-1} = i); \quad i, j \in S;$$

Furthermore, it is assumed that $\{s_t\}$ is independent of $\{u_t\}$. In this way, the (now stochastic) cointegration equation will undergo discrete shifts induced by the values of the Markov chain $\{s_t\}$, with the cointegration vector changing stochastically between $(1; \gamma_1; \gamma_{-0})$ and

$(1; i_1; i_1^-)$; with u_t representing the extent to which the system is out of long-run equilibrium.

This type of model is very flexible, encompassing the regime-shift models discussed by Gregory and Hansen (1996) when $p_{11} = 1$ or $p_{00} = 1$. This specification also allows for a wide range of regime changes, depending on the values of the transition probabilities. In our simulations, the values of the transition probabilities are taken from $(p_{00}; p_{11}) \in \{(0.98; 0.98); (0.9; 0.9); (0.9; 0.5)\}$. We attempt here to experiment different settings for the p_{ij} 's without neglecting their empirical congruence. The first pair of transition probabilities $(p_{00}; p_{11}) = (0.98; 0.98)$ implies highly persistent, almost absorbing regimes, with very few shifts, each regime persisting on average 50 time periods. The pair $(p_{00}; p_{11}) = (0.9; 0.9)$, on the other hand, is less persistent, with an average regime duration of 10 time periods. While the first two pairs allow for symmetry in the persistence of the states, the $(p_{00}; p_{11}) = (0.9; 0.5)$ implies that the second regime is less likely than regime 0, with a mean duration of two time periods, therefore originating a more volatile cointegrating relationship. Concerning the shifts in the coefficients, these are again chosen by the grid $\phi = (\phi_1; \phi_-) \in \{(1; 3); (0.5; 1)\}$. It should be emphasized that both the number and the location of regime shifts are not specified in this DGP.

In all experiments, the number of replications is 2500. In order to attenuate the effect of initial values of the random number generator, $50 + T$ observations are generated in each replication (setting $x_1 = 0$), but the first 50 observations are discarded. Since there are no qualitative differences, we only report the results at the 5% level of significance, although results at the 1% and 10% levels are available upon request.

4 Numerical Results

Given the extension of this study and in order to save space, we have to restrict our attention to some particular experiments and, therefore, only a selection of results is reported here. Tables 1 to 13 display estimates of rejection frequencies of the different tests at the 5% level of significance. The column $\frac{1}{2} = 1$ allows for the estimation of the size of the null-of-no-cointegration (NNC) tests, while representing the "empirical power" of the null-of-cointegration (NC) tests. Conversely, the columns $\frac{1}{2} = 0$ and $\frac{1}{2} = 1$ show the empirical power of NNC tests and size estimates for NC tests. Size-adjusted power is also calculated for NNC tests (in parentheses in the tables), the adjustments being based on the corresponding $\frac{1}{2} = 1$ results of each table.

Before discussing the results for each set of experiments, it is interesting to highlight some general common features in the simulations. First, the performance of NC tests is clearly weak, lacking robustness to parameter non-constancy. Both tests display severe size distortions by rejecting the null of cointegration far more often than they should, especially with changes in the intercept (although larger breaks and autocorrelation reduce, to a small extent, these distortions). On the other hand, the power of the SHI and L_c tests rarely goes beyond 30% and 10%, respectively, except for the DGPs with Markov shifts and less persistent regimes. This evidence suggests that these tests, due to the way they are constructed, tend to behave as structural change tests rather than cointegration tests.

Unlike previous findings in the literature, our experiments reveal that the AEG test generally under-rejects the null of no cointegration, and this conservatism is not dissipated asymptotically (except, again, for Markov changes with more volatile regimes). However, these simulations confirm the conclusion of Gregory and Hansen (1996) regarding their tests, that is, the AEG^a test produces the largest size distortions and $Z_{\text{®}}^a$ is conservative in small samples, while we find that $Z_{\text{®}}$, Z_t and Z_t^a show mild size distortions. It is also clear that, in general, the estimated Type-I error probabilities tend to approach the nominal value of 5% as the sample size increases.

Turning to finite-sample power properties, the size of the break clearly matters here, with larger breaks leading to a considerable loss in power. Furthermore, it appears that all tests have less power when breaks occur both in the intercept and slope, even for small changes in the slope, compared to the case of shifts in the intercept only. In terms of relative (size-corrected) power, $Z_{\text{®}}^a$ and Z_t^a generally perform better than other tests, including AEG^a , and $Z_{\text{®}}$ and Z_t are more powerful than AEG. However, on some occasions with $T = 50$, standard tests (in particular Phillips-Ouliaris tests) seem to do better than Gregory-Hansen tests. Moreover, autocorrelation in the errors ($\frac{1}{2} = 0.5$) affect the power of GH tests to a greater extent than standard tests. Nevertheless, this becomes less problematic as the sample size grows.

Concerning the results for each DGP, Table 1 reports the rejection frequencies when the slope of model (5) follows a random walk. NNC tests are successful in terms of power when the parameter variation is relatively small ($\frac{3}{43} = 0.05$), even with $\frac{1}{2} = 0.5$. If, however, the variance of β_t in (6) is larger ($\frac{3}{43} = 0.2$), power is substantially reduced. Noticeably, asymptotics does not seem to play a role in this case: higher powers are attained when the sample size is 100.

Tables 2 to 5 show the results for the double shift case. The location of the breaks affects the power of NNC tests, but not in a very significant way. Excluding the case of $T = 50$, all

tests perform reasonably well with small changes, but larger breaks worsen power. In contrast, for shifts in the intercept, larger second breaks affect standard tests, while having little effect on GH tests.

For gradual shifts (Tables 6 to 9), the power of standard tests in small samples decreases as the speed of adjustment grows from $\phi = 0.1$ to $\phi = 0.5$. In contrast, the power of GH tests stays very much the same. It is interesting to notice that increasing the sample size is not very relevant in terms of power, particularly for Z_{α} and Z_t tests. In terms of the break location, one cannot distinguish a general pattern of its effects on the set of tests under study. Nevertheless, we may point out that, for instance, the AEG test seems to perform worse with early breaks in the intercept, while AEG and Phillips-Ouliaris tests are less powerful when breaks occur towards the middle and the end of the sample. Overall, and despite this, it appears that the differences in power resulting from different shift locations are not very significant.

Finally, Tables 10 to 13 show the results of the simulations for Markov changes. They reveal that standard NNC cointegration tests, especially Z_{α} and Z_t , perform as well as or better than Gregory-Hansen tests. However, for larger shifts, the size distortions are quite substantial for all NNC tests, increasing with less persistent regimes, namely with $(p_{00}, p_{11}) = (0.9; 0.5)$; and especially for changes in the intercept and slope. Not unexpectedly, these distortions are not attenuated asymptotically, in fact worsening for some tests. This may be explained by the fact that the number of breaks in the sample does not remain constant, increasing as the sample size increases, even for more persistent regimes. As said before, NC tests are more powerful and more robust in this context of Markov changes and more volatile regimes, but still not very successful with the smaller sample sizes.

5 Conclusion

In this paper, we have investigated the finite-sample properties of cointegration tests when the cointegration vector is subject to regime shifts. In our experiments, we have found that tests with no cointegration as the null hypothesis have reasonable power for a range of different parameter shifts. In particular, the Z_t^p test seems to be the most well-balanced test among all others, in terms of size and power, which confirms the findings of Gregory and Hansen (1996) in a less broad context. However, it is clear that the size of the breaks is a determinant factor for power, which decreases for larger shifts, as well as the number of shifts, attending to the results with

Markov changes and regimes of shorter duration. On the other hand, the relative location of the break in the sample appears to be less significant. Our experiments also reveal that the use of tests with the null of cointegration should be avoided, given their discouraging performance in terms of power and size.

Recent empirical research shows that it is relevant to consider structural changes in many univariate and multivariate non-stationary time series (see some of the works cited before). Although the majority of these works concentrates on detecting breaks, we may also find some attempts to model economic relationships subject to parametric shifts, in a cointegration framework (see Hall et al., 1997, Krolzig, 1997 and Hansen, 1999). Notwithstanding this, an appropriate empirical modelling strategy accounting for structural changes is yet to be defined. This paper sought to contribute further to this discussion.

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