Oil and the Macroeconomy: using wavelets to analyze old issues*

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Abstract

We use (cross) wavelet analysis to decompose the time-frequency effects of oil price changes on the macroeconomy. We argue that the relation between oil prices and industrial production is not clear-cut. There are periods and frequencies where the causality runs from one variable to the other and vice-versa, justifying some instability in the empirical evidence about the macroeconomic effects of oil price shocks. We also show that the volatility of both the inflation rate and the industrial output growth rate started to decrease in the decades of 1950 and 1960.

Keywords: Business cycles, oil shocks, wavelets, cross wavelets, wavelet coherency.

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1 Introduction

Hamilton (1983, 1985), Gisser and Goodwin (1986), Aguiar-Conraria and Wen (2007) and many others[1] provide empirical evidence that until the mid-1980s oil prices were a significant determinant of U.S. economic activity. Although Hooker (1996) argued that the correlation between oil prices and economic activity is much less clear after 1985, more recently, other approaches have confirmed the robustness of previous results. For example, Kilian (2008a) looks at historical accounts and industry sources to identify exogenous oil production shortfalls, Baumeister and Peersman (2008), in a Bayesian VAR framework, use sign restrictions to disentangle supply from demand oil price shocks, and Cavallo and Wu (2006) construct an oil shocks measure based on news exogenous to the U.S. economy. Once these more realistic identification methods are considered, the basic results obtained for the 1970s and 1980s are replicated.

In the cited works, the analysis is exclusively done in the time-domain. The frequency-domain is left out. However, some interesting relations may exist at different frequencies: oil prices may act like a supply shock at high and medium frequencies, therefore affecting industrial production, while, in the longer run (lower frequencies) it is the industrial production, through a demand effect, that affects oil prices.

To uncover relations at different frequencies, it is common to utilize Fourier analysis. However, under the Fourier transform, the time information is completely lost, making it difficult to distinguish transient relations or to identify structural changes. To include the time dimension, Gabor (1946) introduced the Short Time Fourier Transform. The basic idea is to break a time-series into smaller sub-samples and apply the Fourier transform to each sub-sample. However, this approach is inefficient because the frequency resolution is the same across all different frequencies. Actually, one major advantage afforded by the wavelet transform is the ability to perform natural local analysis of a time-series in the sense that the length of wavelets varies endogenously: it stretches into a long wavelet

[1] See Kilian (2008b) for a thorough review.

function to measure the low frequency movements; and it compresses into a short wavelet function to measure the high frequency movements. See Hogan and Lakey (2005) for a discussion and comparison between time-frequency and time-scale (wavelets) methods.

We use wavelets to analyze the impact of oil price changes in two macroeconomic variables: industrial production and inflation. Following Jevrejeva et al. (2003), Bloomfield et al. (2004), and Cazelles et al. (2007), three tools are utilized: the wavelet power spectrum, wavelet coherency and wavelet phase-difference. While the wavelet power spectrum describes the evolution of the variance of a time-series at the different frequencies, the wavelet coherency can be seen as a localized correlation coefficient in the time-frequency space. The phase-difference gives us information on the delay between the oscillations of two time-series. We refer the reader to Cazelles et al. (2007) and to Aguiar-Conraria et al. (2008) for a detailed exposition of these tools and their properties.

2 Wavelet analysis

Wavelet theory was born in the mid-1980s (Grossmann and Morlet 1984, Goupillaud et al. 1984). After 1990, the literature rapidly expanded to several disciplines, such as physics or epidemiology. Unfortunately, this technique is infrequently used in economics. The work of Ramsey and Lampart (1998a and 1998b), and Gençay et al. (2001a and 2001b) is unknown to the majority of economists. Among the exceptions to this rule, one can point to Connor and Rossiter (2005), Gençay et al. (2005), and Gallegati and Gallegati (2007). See Crowley (2007), for a recent survey of wavelet applications to economic data.

Probably, wavelets are not more popular among economists, because they have been applied either to analyze individual time-series (Gallegati and Gallegati 2007) or to individually analyze several time-series (one each time), whose decompositions are then studied using traditional time-domain methods (Ramsey and Lampart 1998a and 1998b). Recently, Gallegati (2008) — using the Maximum Overlap Discrete Wavelet Transform —

and Crowley and Mayes (2008) and Aguiar-Conraria et al. (2008) — using the Continuous Wavelet Transform — showed how the cross-wavelet analysis could be fruitfully used to uncover time-frequency interactions between two economic time-series. Still, most surely, wavelets will not become very popular in economics until a concept analogous to the spectral partial-coherence is developed. On this regard, we see multivariate spectral analysis using Hilbert wavelet pairs — see Crowley et al. (2006) — and SLEX (smooth localized complex exponentials) Analysis of Multivariate Nonstationary Time Series — see Ombao et al. (2005) — as promising venues. Also, see Ombao and Vam Bellegem (2008) for a discussion on other approaches to estimate time-varying coherences.

2.1 Continuous Wavelet Transform

The continuous wavelet transform, with respect to the wavelet ψ, is a function $W_x(s,\tau)$ defined as:

$$W_x(s,\tau) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{s}} \psi^* \left(\frac{t-\tau}{s} \right) dt, \quad (1)$$

where * denotes complex conjugation. The parameter s is a scaling factor that controls the length of the wavelet and τ is a location parameter that indicates where the wavelet is centered. Scaling a wavelet simply means stretching it (if $|s| > 1$), or compressing it (if $|s| < 1$).

If the wavelet function $\psi(t)$ is complex, the wavelet transform W_x will also be complex. The transform can then be divided into the real part ($\mathcal{R}\{W_x\}$) and imaginary part ($I\{W_x\}$), or amplitude, $|W_x|$, and phase, $\tan^{-1}\left(\frac{\mathcal{I}\{W_x\}}{\mathcal{R}\{W_x\}}\right)$. The phase of a given time-series $x(t)$ is parameterized in radians, ranging from $-\pi$ to π. In order to separate the phase and amplitude information of a time series it is important to make use of complex wavelets. Just like with the Fourier transform, under some regularity conditions, we can reconstruct $x(t)$ from its continuous wavelet transform.

2.2 The Morlet wavelet

The minimum requirements imposed on a function $\psi(t)$ to qualify for being a *mother (admissible or analyzing) wavelet* are that $\psi(t)$ is a square integrable function and satisfies the *admissibility condition*: $\int_{-\infty}^{\infty} \frac{|\Psi(f)|}{|f|} df < \infty$, where $\Psi(f)$ is the Fourier transform of ψ. For most purposes, the admissibility condition is equivalent to requiring $\int_{-\infty}^{\infty} \psi(t)\, dt = 0$. Therefore, ψ has to wiggle up and down the $t-$axis, behaving like a wave, justifying the choice of the term wavelet.

There are several wavelet functions available, such as Morlet, Mexican hat, Daubechies, etc. The choice depends on the particular application one has in mind. We choose a complex wavelet as it yields information on the amplitude and phase, both essential to study synchronism between different time-series.

An important property of a wavelet function is its accuracy. Define the center of the wavelet ψ by $\mu_t = \int_{-\infty}^{\infty} t\, |\psi(t)|^2\, dt$ and the variance by $\sigma_t^2 = \int_{-\infty}^{\infty} (t-\mu_t)^2\, |\psi(t)|^2\, dt$. Similarly, define the center μ_f and variance σ_f of the Fourier transform of ψ. The interval $[\mu_t - \sigma_t, \mu_t + \sigma_t]$ is the set where ψ attains its "most significant" values whilst the interval $[\mu_f - \sigma_f, \mu_f + \sigma_f]$ plays the same role for Ψ. The rectangle $[\mu_t - \sigma_t, \mu_t + \sigma_t] \times [\mu_f - \sigma_f, \mu_f + \sigma_f]$ in the $(t, f)-$plane is the Heisenberg box in the time-frequency plane. We say that ψ is localized around the point (μ_t, μ_f) of the time-frequency plane with uncertainty given by $\sigma_t \sigma_f$. In our context, the Heisenberg's principle establishes that $\sigma_t \sigma_f \geq \frac{1}{4\pi}$.

The Morlet wavelet,

$$\psi(t) = \pi^{-\frac{1}{4}} \exp(i\omega_0 t) \exp\left(-\frac{1}{2}t^2\right), \tag{2}$$

is a complex valued wavelet with optimal joint time-frequency concentration, in the sense that it reaches the lower bound, $\sigma_t \sigma_f = \frac{1}{4\pi}$. Choosing $\omega_0 = 6$, the wavelet scale, s, is inversely related to the frequency, $f \approx \frac{1}{s}$, simplifying the interpretation of the wavelet analysis.

2.3 The wavelet power spectrum

Typically one has to deal with a discrete time-series $\{x_n,\ n = 0, ..., N-1\}$ of N observations with a uniform time step δt and the integral in (1) is then discretized:

$$W_m^x(s) = \frac{\delta t}{\sqrt{s}} \sum_{n=0}^{N-1} x_n \psi^* \left((n-m) \frac{\delta t}{s} \right), \qquad m = 0, 1, \ldots, N-1. \tag{3}$$

Although it is possible to calculate the wavelet transform using the above formula for each value of s and m, one can also identify the computation for all the values of m simultaneously as a convolution of two sequences. The standard procedure is to calculate this convolution as a simple product in the Fourier domain, using the Fast Fourier Transform algorithm to go forth and back from time to spectral domain. As with other types of transforms, the CWT applied to a finite length time-series inevitably suffers from border distortions, which increase with s. The region in which the transform suffers from these edge effects is called the cone of influence. In this area, the results should be interpreted carefully, as they may be affected by the zero padding at the beginning and the end of the time series.

The wavelet power spectrum is simply $|W_n^x|^2$. The wavelet power spectrum characterizes the distribution of the energy (spectral density) of a time series across the two-dimensional time-scale plane, leading to a time-scale (or time-frequency) representation.

Although Torrence and Compo (1998) have shown how the statistical significance of wavelet power can be assessed against the null hypothesis that the data generating process is given by an $AR(0)$ or $AR(1)$ stationary process with a certain background power spectrum (P_f),[2] for more general processes one has to rely on Monte-Carlo simulations. We

[2] $D\left(\frac{|W_x(s,\tau)|^2}{\sigma_x^2} < p\right) = \frac{1}{2} P_f \chi_v^2(p)$, at each time τ and scale s. The value of P_f is the mean spectrum at the Fourier frequency f that corresponds to the wavelet scale s — in our case $s \approx \frac{1}{f}$ — and v is equal to 1 or 2, for real or complex wavelets respectively.

assess the statistical significance of the wavelet power against the null hypotheses that each variable follows an $ARMA(p,q)$ process, with no pre-conditions on p and q. The simulations are done using the amplitude adjusted Fourier-transformed surrogates proposed by Schreiber and Schmitz (1996).

2.4 Wavelet coherency and phase difference

The cross wavelet transform of two time series, $x = \{x_n\}$ and $y = \{y_n\}$, is defined as $W_n^{xy} = W_n^x W_n^{y*}$. The cross wavelet power is given by $|W_n^{xy}|$. While the wavelet power spectrum depicts the variance of a time series, with times of large variance showing large power, the cross–wavelet power of two time series depicts the covariance between these time series at each scale or frequency. Wavelet coherency is the ratio of the cross-spectrum to the product of the spectrum of each series, and can be thought of as the local (both in time and frequency) correlation between two time-series:

$$R_n(s) = \frac{\left|S\left(s^{-1}W_n^{xy}(s)\right)\right|}{S\left(s^{-1}\left|W_n^x\right|\right)^{\frac{1}{2}} S\left(s^{-1}\left|W_n^y\right|\right)^{\frac{1}{2}}}, \tag{4}$$

where S denotes a smoothing operator in both time and scale – see Aguiar-Conraria et al. (2008) for details. Again, to assess the statistical significance of the estimated wavelet coherency, we follow Schreiber and Schmitz (1996).

We focus on the wavelet coherency, instead of the wavelet cross spectrum for two reasons: (1) the wavelet coherency has the advantage of being normalized by the power spectrum of the two time-series, and (2) Maraun and Kurths (2004) show that the wavelet cross spectrum can show strong peaks even for the realization of independent processes suggesting the possibility of spurious significance tests.

The phase of a given time-series can be viewed as the position in the pseudo-cycle of the series. The phase-difference describes the relative positions of the two time series:

$$\phi_{x,y} = \tan^{-1}\left(\frac{\mathcal{I}\left\{W_n^{xy}\right\}}{\mathcal{R}\left\{W_n^{xy}\right\}}\right), \qquad \text{with } \phi_{x,y} \in [-\pi, \pi]. \tag{5}$$

A phase-difference of zero indicates that the time-series move together at the specified frequency. If $\phi_{x,y} \in \left(0, \frac{\pi}{2}\right)$ then the series move in phase, but the time-series y leads x. If $\phi_{x,y} \in \left(-\frac{\pi}{2}, 0\right)$ then it is x that is leading. A phase-difference of π (or $-\pi$) indicates an anti-phase relation. If $\phi_{x,y} \in \left(\frac{\pi}{2}, \pi\right)$ then x is leading. Time-series y is leading if $\phi_{x,y} \in \left(-\pi, -\frac{\pi}{2}\right)$.

3 Data analysis[3]

In Figure 1, we can see the estimated power spectrum for several monthly time series for the United States economy: Oil Prices (growth rate), Industrial Production Index (growth rate) and Inflation (based on the Consumer Price Index). All the data is from the Federal Reserve Bank of St. Louis and runs from the beginning of 1946 until the end of 2007.

It is clear that the different time series have different characteristics in the time-frequency domain. During the late 1940s and early 1950s, the inflation rate variance was quite high both at low and high scales. It decreased in the 1960s and in the 1970s and 1980s, probably as a consequence of very active oil shocks, the variance of the inflation rate became higher, but in this case, the effect is clearer at medium and high scales, suggesting that we were facing medium and long term shocks to inflation. Note that although the boundaries of the sample are affected by the edge effects, these effects should decrease volatility, not increase it (just notice that the edge effects appear because we add a series of zeros, whose variance is, obviously, null). Therefore, it is a fair conclusion to say that the 1940s and early 1950s is a high powered region.

[3] We thank Bernard Cazelles for providing us the code used in Cazelles et al. (2007). The code is available at request. Alternatively, the code of Jevrejeva, Grinsted and Moore can be found at http://www.pol.ac.uk/home/research/waveletcoherence/.

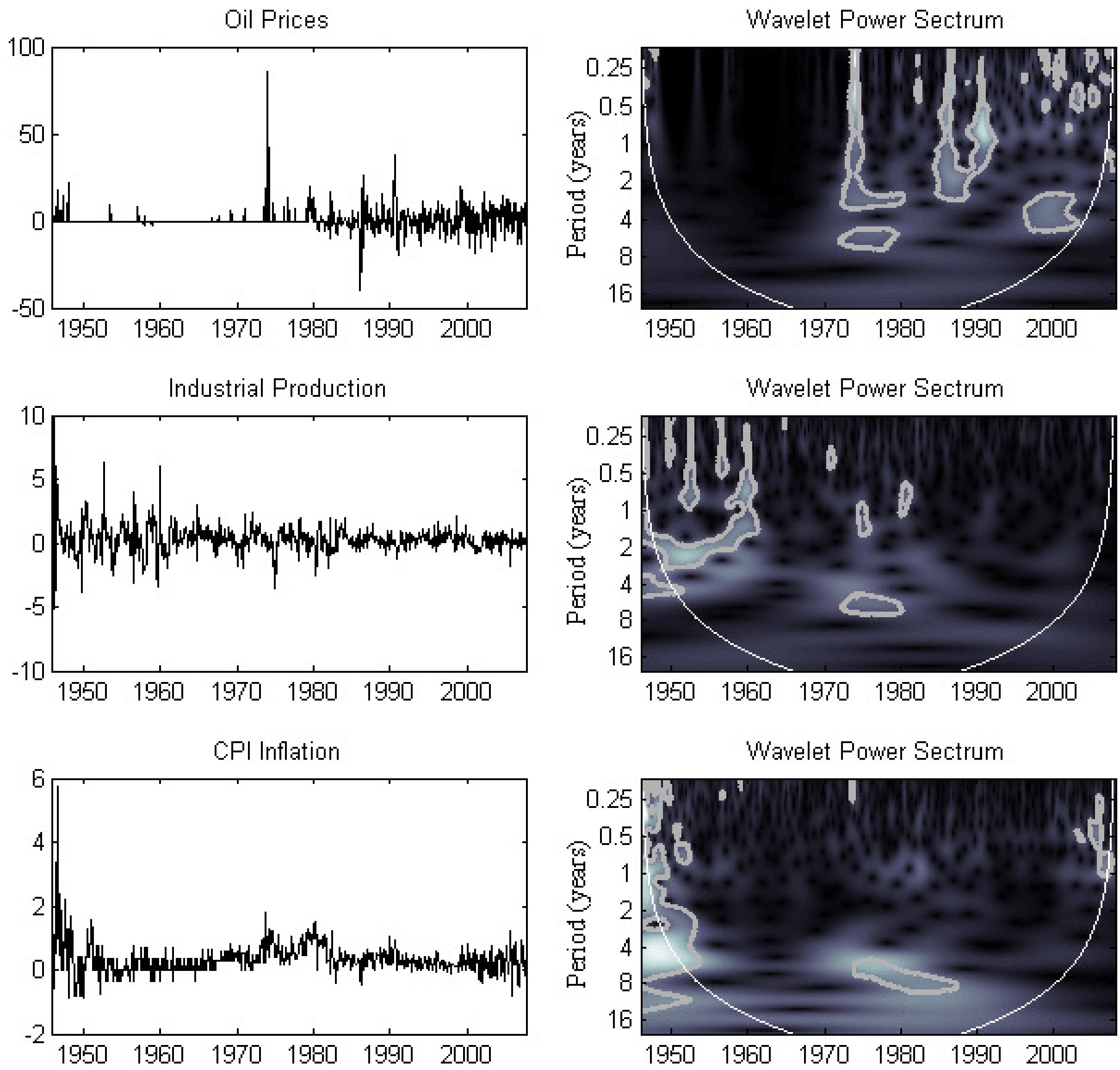

Figure 1: On the left: Time series plots. On the right: Wavelet Power Spectrum — The thick gray contour designates the 5% significance level estimated by Monte Carlo simulations (500 trials). The cone of influence, which indicates the area affected by edge effects, is the outside region of the white line. The code for power ranges from dark gray (low power) to light gray (high power).

The power, at all scales, of the industrial production was quite high until 1950s. After that, it has been steadily decreasing, with an exception between mid 1970s and mid 1980s, when the variance at the business cycle frequency (3 to 8 years) was quite high. It has become common in the literature to argue that we have been observing, in the last two decades, a decrease in the volatility of GDP in the United States. After World War II, the volatility was quite high at business cycle frequencies. In the 1960s, the volatility decreased at all scales, suggesting that the "Great Moderation" started in the decade of 1960. Then macroeconomic volatility increased again, probably due to the oil shocks, at the business cycle frequency in the 1970s, however this increase was temporary. Our results are absolutely in line with Blanchard and Simon (2001), who also argued that macroeconomic volatility has been decreasing since World War II (although temporarily revived in the 1970s and early 1980s).

If we look at the power spectrum of the oil prices growth rate, we observe that until mid-1970s these were very stable. Between 1975 and 1980, the $\frac{1}{12} \sim 6$ years timescale band shows high power. We observe similar effects in the late 1980s and early 1990s, and again in 2000. A structural change occurred in the oil price series in the mid 1970s, after which oil prices became much more volatile.

In Figure 2, we estimate the coherency between the oil price series and industrial production and between oil prices and inflation. We also estimate the phase of the oscillations, as well as their phase-difference (same picture, on the right). Given that for oil prices and industrial production the most coherent regions are between the $4 \sim 12$ years band, we focus our phase difference analysis on two frequency bands: $4 \sim 8$ and $8 \sim 12$ years. For the same reasons, when analyzing the phases and phase-differences between oil prices and inflation, we focus on three frequency bands: $4 \sim 8$, $8 \sim 12$ and $12 \sim 16$ years. For each band, we calculate the average phase and phase-difference.

Many interesting structural changes have occurred in the relationship between oil prices and industrial production. Between the early 1960s and late 1970s, we have a high co-

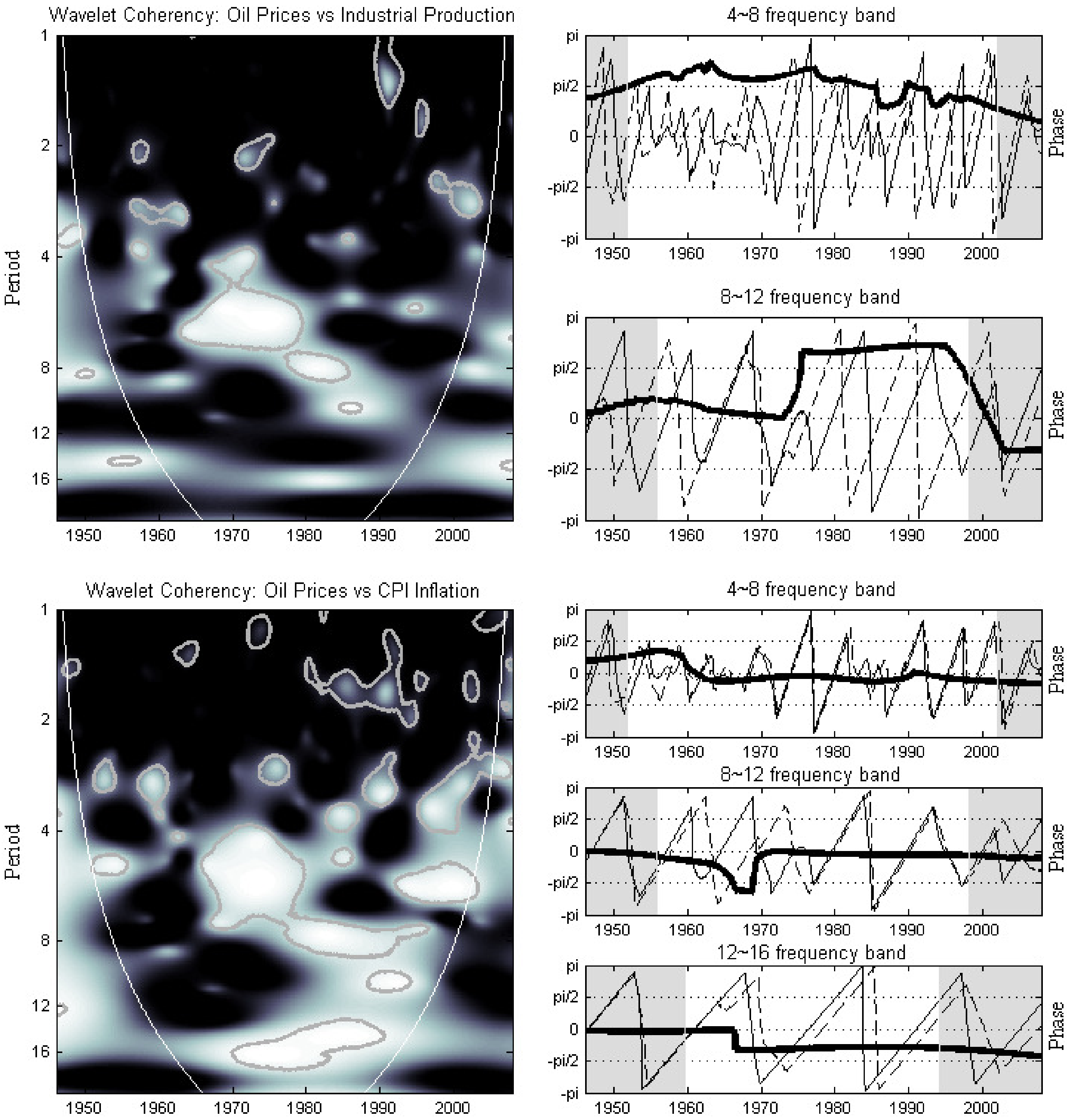

Figure 2: On the Left:Cross-Wavelet Coherency − The gray thick contour designates the 5% significance level estimated by Monte Carlo simulations (500 trials). The cone of influence, which indicates the area affected by edge effects, is the outside region of the white line. The code for coherency ranges from dark gray (low coherency – close to zero) to light gray (high coherency – close to one). On the right: Phase and phase-difference between the two series. The solid line represents the oil price series phase, and the dashed line represents the phase of the other variable. The thick black line gives us the phase-difference between the two series. The shaded area is the area affected by edge effects.

herency region at business cycle frequencies ($4 \sim 8$ years). During the 1980s, the high coherency region shifted to the $8 \sim 12$ year band. The phase-difference gives us more details. Looking first at the $4 \sim 8$ years frequency band, we observe that until the mid-1980s the phase-difference is consistently between $\frac{\pi}{2}$ and π. After that it stays for most of the time between 0 and $\frac{\pi}{2}$. Therefore, at these timescales, until the mid-1980s we had a negative relation between oil prices and industrial production with oil prices leading. After 1985, the series became in-phase, with industrial production leading. These results suggest that, in this frequency band, oil price changes that have occurred after mid 1980s were demand-driven, while, until that period, oil-price increases led to decreases in the industrial production, capturing the negative effects of oil price shocks that were highlighted in the empirical literature until 1985. Our results are compatible with the results of several authors — e.g., see Mork (1989), Hooker (1996) and Hamilton (2003) —, who have identified a structural break in the relation between oil prices and the Macroeconomy at around 1985.

If we focus our attention on the $8 \sim 12$ year frequency band, it is clear that the oil crisis in 1974 had a significant impact. After the mid-1970s, we observe not only an increase the the series coherency but we also observe that the series ceased to be in phase until the mid-1990s. This suggests that the negative effects of oil price shocks shifted to lower frequencies. After the end of 1995, we observe another change that helps to explain some recent results: in this frequency band, oil price changes were demand-driven. Putting everything together, one concludes that demand driven oil price shocks have become more important after the mid-1990s, lending further support to the conclusions of Kilian (2009) and Baumeister and Peersman (2008).

In Figure 2, we also see that the relation between oil prices and inflation is stronger and more stable (and almost trivial). The phase differences reveal a very stable relation. At most scales and for most of the time, the phase difference has consistently been between zero and $-\pi/2$. This suggests that oil price increases lead the consumer price index increases. Looking at coherency some different patterns emerge. There is a structural change in the late 1960s. Before that, there were not many periods of high coherency. In the 1970s there is high coherency at both medium ($4 \sim 8$ years band) and large scales ($12 \sim 16$ years band). During the 1980s decade, we observe high coherency in the $8 \sim 16$ years band. After 1990, only at very high scales do we observe strong coherency. Some political economy major events that happened during these decades may explain this evolution. The decade of 1970 is the decade of major oil shocks. After that, in 1980, there was a strong shift in American monetary policy. In July 1979, Paul Volcker was nominated the Chairman of the Federal Reserve Board. Volcker announced a fierce fight against inflation and implemented a very restrictive monetary policy as a reaction to the inflationary pressures of the second oil shock. In 1987, and during the entire decade of 1990, when Alan Greenspan was the chairman of the Federal Reserve, inflation was under control.

4 Conclusion

This paper used wavelet analysis to study the relationship between oil price increases and inflation. This relation was very stable and confirmed some common ideas. Namely, oil price increases lead inflation increases across time and across frequencies. We could also see that the tight monetary policy of the 1980s proved to be successful, with a decrease of the inflationary impact of oil price shocks. During the 1990s, the inflationary impacts of oil price increases were also very well contained.

We also studied the relationship between oil and output, and uncovered interesting patterns as well as three structural breaks. The first structural break occurred in the mid-1970s, after the first oil crisis: around that period, the region of high coherency shifted from the $4 \sim 8$ to the $8 \sim 12$ year frequency band. This observation, together with the changes in the phase-difference, suggests that the negative effects of oil price shocks shifted to lower frequencies. The second structural break, first identified by Mork

(1989), occurred in the mid-1980s: at the $4 \sim 8$ frequency band, the series became in-phase, with industrial production leading. This shift suggests that, in this frequency band, oil price changes that occurred after mid 1980s were more demand-driven. Finally, and again focusing our attention on the $8 \sim 12$ year frequency band, after the end of 1995, we observe another change that substantiates some recent empirical results: The phase analysis is compatible with the suggestion that oil price changes that have occurred after the mid-1990s were demand-driven. These results are broadly in line with those of Kilian (2009) and also with Baumeister and Peersman (2008), who concluded that, approximately since the Asian crisis, demand-driven oil price shocks have become more important than supply-side induced shocks. Putting all this information together, one concludes that demand driven oil price shocks became important around 1985, and became even more important after some period around 1995.

Finally, our results lend further support to the conclusions of Blanchard and Simon (2001) about the volatility of both the inflation and the output growth. Macroeconomic volatility started to decrease in the decades of 1950 and 1960, suggesting that the great moderation started then, but that it was temporarily interrupted due to the oil crises of the 1970s, whose effects were felt until the mid-1980s.

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