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Complementarities and Costly Investment in a One-Sector Growth Model

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ABSTRACT

The presence of complementarities generally makes a growth model nonlinear, hence delivering multiple equilibria. Introducing internal investment costs in the R&Dbased growth literature, we develop a growth model which combines the assumptions of complementarities between capital goods in the production function and of internal costly investment in capital. We find that with such combination of complementarities and costly investment, the growth model delivers a single equilibrium.

 $Keywords\ and\ Phrases:$ Complementarities, Costly Investment, Economic Growth

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1 Introduction

With this paper, we propose a growth model which combines the assumption of complementarities between capital goods in the production function with the assumption of internal costly investment in capital.

Bryant (1983) stresses the importance of complementarities between capital goods in production. The presence of complementarities means that if the number of its complementary goods increases, the production of a capital good will increase. In turn, by increasing its output, a producer of a capital good is raising the demand for its complementary goods. Personal computers, printers and communication networks are examples of capital goods that are complements.

Additionally, as Romer (1996) writes, investment decisions are better captured by a standard theory which emphasizes the existence of costs to accumulating capital. The assumption of internal costs of investment, that we introduce, is new to the R&D-based growth literature.

As analysed by Matsuyama (1995, 1997), the presence of complementarities in a growth model generally makes it nonlinear, hence generating multiple equilibria. We find that, in our developed model, the presence of complementarities combined with the existence of costs to investment generates a single equilibrium.

Our proposed model builds on the multiple equilibria model by Evans, Honkapohja and Romer (1998) in assuming complementarities between capital goods in the production function. However, we modify their two-sector structure into a one-sector framework and replace Evans et al.'s (1998) analytically-nonobservable external cost of investment with an analytically-observable internal investment cost function due to Hayashi (1982).

Our main finding is that, while the presence of complementarities in a growth model generally makes it deliver multiple equilibria, in our model combining the assumption of complementarities with the assumption of internal costly investment allows the model to generate a unique equilibrium.

The paper is organised as follows. After this Introduction, Section 2 provides motivation for the introduction of internal costs of investment. Section 3 presents the specification of our proposed general equilibrium growth model, and its main results. This research is closed with Concluding Remarks.

2 Motivation for Internal Costly Investment

Consider the baseline model of investment in which firms maximise the present discounted value of their cash flows, facing zero capital investment costs. Assume also that capital depreciation is zero, for simplicity. The *current-value* Hamiltonian is, then:

$$H(t) = F(K(t), \overline{L}) - I(t) + q(t)(I(t) - K(t)),$$

where $F(K(t), \overline{L})$ is the production function, I(t) represents investment, K(t)is capital accumulation and q is the *current-value* of capital.

The solution to this maximisation problem is the standard condition:

$$\frac{dF(K,\overline{L})}{dK} = r$$

As Hayashi (1982) analyses, in this model, for a given level of output and a linearly homogeneous production function, the optimal level of capital stock can be determined, but the rate of optimal investment is indeterminate. This means that if, for instance, the initial level of capital K(0) is lower than the optimal capital level K^* , investment will be infinitely positive. Or, if the interest rate falls, the stock of capital that satisfies the standard condition increases, and this requires an infinite rate of investment. However, as it is limited by aggregate output, investment cannot be infinite.

This indeterminacy of investment led to modifications of the baseline model, which involve the introduction of costs to the accumulation of capital. Hence, according to the modified neoclassical investment theory, the representative firm maximises the present discounted value of its cash flows, subject to capital installation costs.

The specification for the capital installation cost function that we use in this paper is an application of Hayashi's (1982) cost of investment framework to a continuous time context, as done by Benavie et al. (1996), Cohen (1993) and Van Der Ploeg (1996), in models different from the one developed in this paper.

According to this investment cost specification, installing I(t) = K(t) new units of capital requires the firms to spend an amount given by:

$$J(t) = I(t) + \frac{1}{2}\theta \frac{I(t)^2}{K(t)}$$

where the installation cost is $C(I(t), K(t)) = \frac{1}{2} \theta \frac{I(t)^2}{K(t)}$

The *current-value Hamiltonian* is, then:

$$H(t) = F(K(t), \overline{L}) - I(t) - \frac{1}{2}\theta \frac{I(t)^2}{K(t)} + q(t)(I(t) - K(t))$$

A one-unit increase in the firm's capital stock increases the present value of the firm's cash flow by q, and thus increases the value of the firm by q. Hence q is the market value of a unit of capital.

As the purchase price of capital is assumed to be $P_K = 1$, the ratio of the market value of a unit of capital to its replacement cost, $\frac{q}{P_{\kappa}}$, is equal to q. This ratio is known as Tobin's (1969) marginal q.

In turn, the ratio of the market value of the firm to the replacement cost of its total capital stock, $\frac{V}{P_K K} = \frac{V}{K}$ is called *average q*. It is *marginal q* that is relevant to investment. However only *average q* is

observable. Empirical studies have thus relied on average q as an approximation

to marginal q. Hayashi (1982) solved this empirical issue because with his installation cost function, described above, marginal q and average q are equal.

After this motivation for the capital installation cost function adopted for our proposed growth model, we proceed with the specification of the model and its main results.

3 Specification and Results of the Model

3.1 Consumption Side

The preferences structure adopted is the standard optimising one. Infinitely lived homogeneous consumers maximise, subject to a budget constraint, the discounted value of their representative utility:

$$\max_{C(t)} \int_0^\infty e^{-\rho t} \frac{C(t)^{1-\sigma}}{1-\sigma} dt$$

s.t.
$$B(t) = rB(t) + w(t) - C(t)$$

where variable C(t) is consumption in period t, ρ is the rate of time preference and $\frac{1}{\sigma}$ is the elasticity of substitution between consumption at two periods in time. Variable B(t) stands for total assets, r is the interest rate, w(t) is the wage rate, and it is assumed that households provide one unit of labour per unit of time.

Consumption decisions are then given by the familiar Euler equation:

$$g_c = \frac{\dot{C}}{C} = \frac{1}{\sigma}(r - \rho), \qquad (1)$$

which tells us that a balanced growth path solution requires a constant interest rate.

3.2 Production Side

The production side is composed by three productive activities: final good production, capital goods production and invention of new capital goods, that is, research and development (R&D) activities.

The technology in this economy is characterised by a combination of the effects of complementarities between capital goods in the production function and the effects of internal costly investment in capital.

Our proposed framework builds on Evans, Honkapohja and Romer's (1998) model in assuming complementarities between capital goods in the production function.

Our model is structurally distinct from Evans et al.'s (1998) model in three key aspects. Firstly, Evans et al.'s model is a two-sector model, having a

non-linear trade-off between consumption and investment in a general-purposecapital which is composed by physical capital and inventions.

In comparison, our model has a one-sector structure, in which the same technology is used for consumption, investment in physical capital and production of new designs, as in Rivera-Batiz and Romer (1991).

Secondly, in Evans et al.'s (1998) model the cost of investment is external, whereas the model we develop assumes an internal cost of investment, in which final-good producers incur an internal investment cost when accumulating capital.

Thirdly, Evans et al. (1998) obtain nonlinearity through an analyticallynon-observable mechanism, as the price of general-purpose-capital in terms of consumption varies positively with the growth rate, through an analyticallynon-observable function.

In contrast, as our introduced investment cost function is analytically-observable, the nonlinearity of the model can be analytically observed.

Let us proceed, then, with the description of the model.

3.2.1 Complementarities between Capital Goods

For the specification of final good production activities, we build on Evans et al. (1998) in assuming that the final good Y is produced using as inputs labour L, assumed constant, and a number A of differentiated durable capital goods i, each produced in quantity x_i . Capital goods enter complementarily in the production function. All this is captured by the following production function:

$$Y(t) = L(t)^{1-\alpha} \left(\int_0^{A(t)} x_i(t)^{\gamma} di \right)^{\phi} \qquad , \qquad \phi > 1 \qquad , \qquad \gamma \phi = \alpha, \qquad (2)$$

where the assumption $\phi > 1$ is made so that capital goods are complementary to one another, that is, so that an increase in the quantity of one good increases the marginal productivity of the other capital goods. The restriction $\gamma \phi = \alpha$ is imposed in order to preserve homogeneity of degree one¹.

The second productive activity concerns the production of the physical machines for each of the already invented types of capital goods. Assuming that it takes one unit of physical capital to produce one physical unit of any type of capital good, in each period physical capital K is related to the capital goods by the rule:

$$K(t) = \int_0^{A(t)} x_i(t) di$$

Turning now to the R&D activities, new designs are invented with the same technology as that of the production of the final good and of capital goods. We assume that the invention of patent *i* requires $P_A i^{\xi}$ units of foregone output,

 $^{^1 \}mathrm{See}$ Jones (2007) for a different specification of complementarity, through a CES production function.

where P_A is the fixed price of one new design in units of foregone output, and i^{ξ} represents an additional cost of patent *i* in terms of foregone output, meaning that there is a higher cost for designing goods with a higher index. This extra cost is introduced in order to avoid a explosive growth.

Total investment in each period W(t) is then given by:

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$$W(t) = K(t) + P_A A(t) A(t)^{\xi}, \qquad (3)$$

where K(t) represents investment in physical capital, and $P_A A(t) A(t)^{\xi}$ represents investment in the invention of new designs.

Total capital W is equal to:

$$W(t) = K(t) + P_A \frac{A(t)^{\xi+1}}{\xi+1},$$
(4)

and it accumulates according to:

$$W(t) = Y(t) - C(t) \tag{5}$$

As in Evans et al. (1998), in order to solve the model for a constant growth rate, a parameter restriction is imposed:

$$\xi = \frac{\phi - 1}{1 - \alpha}$$

Final good producers are price takers in the market for capital goods. In equilibrium they equate the rental rate on each capital good with its marginal productivity. So the demand curve faced by each capital good producer is:

$$R_{j}(t) = \frac{dY(t)}{dx_{j}(t)} = \phi \gamma L^{1-\alpha} x_{j}(t)^{\gamma-1} \left(\int_{0}^{A(t)} x_{i}(t)^{\gamma} di \right)^{\phi-1},$$
(6)

which is equivalent to:

$$x_j(t) = \left[\frac{\alpha L^{1-\alpha} \left(\int_0^{A(t)} x_i(t)^{\gamma} di\right)^{\phi-1}}{R_j(t)}\right]^{\frac{1}{1-\gamma}}$$

Capital good firms face the same market conditions. So they produce the same quantities of their differentiated goods and sell them at the same price. That is, the symmetry of the model implies that $R_j(t) = R(t)$, and $x_j(t) = x(t)$. Hence the production function for aggregate output 2, can be rewritten as:

$$Y = L^{1-\alpha} A^{\phi} x^{\alpha}$$
$$= L A^{1+\xi} \left(\frac{\alpha}{R}\right)^{\frac{\alpha}{1-\alpha}},$$

Likewise, the expression for total capital 4, can be rewritten as:

$$W = A^{1+\xi} \left[L \left(\frac{\alpha}{R}\right)^{\frac{1}{1-\alpha}} + \frac{P_A}{\xi+1} \right]$$

It follows that:

$$\frac{Y}{W} = \frac{L\left(\frac{\alpha\gamma}{r}\right)^{\frac{\alpha}{1-\alpha}}}{\left[L\left(\frac{\alpha\gamma}{r}\right)^{\frac{1}{1-\alpha}} + \frac{P_A}{\xi+1}\right]} = B,$$

that is, the production function of this economy can be expressed as a function of total capital in the following way:

$$Y = BW,$$

where B, the marginal productivity of total capital, is constant.

3.2.2 Internal Costly Investment

Final good producers own total capital W and incur an internal investment cost. We assume that installing I(t) = W(t) new units of total capital requires the final good firms to spend an amount given by:

$$J(t) = I(t) + \frac{1}{2}\theta \frac{I(t)^2}{W(t)},$$

where $C(I(t), W(t)) = \frac{1}{2} \theta \frac{I(t)^2}{W(t)}$ represents the Hayashi's (1982) installation cost. Final good firms choose their investment rate so as to maximise the present

Final good firms choose their investment rate so as to maximise the present discounted value of their cash flows. Their profit maximisation problem is then:

$$\max_{I(t)} V(t) = \int_0^\infty \left(Y(t) - I(t) - \frac{1}{2} \theta \frac{I(t)^2}{W(t)} \right) e^{-rt} dt$$

s.t.
$$W(t) = I(t)$$

The current-value Hamiltonian is:

$$H(t) = Y(t) - I(t) - \frac{1}{2}\theta \frac{I(t)^2}{W(t)} + q(t)(I(t) - W(t)),$$

where q(t) is the market value of capital.

The transversality condition of this optimization problem is $\lim_{t\to\infty} e^{-rt}q(t)W(t) = 0$, the first-order condition is equivalent to:

$$\frac{I}{W} = \frac{q-1}{\theta},\tag{7}$$

and the co-state equation is equivalent to:

$$\frac{\dot{q}}{q} = r - \frac{B + \frac{1}{2}\theta \left(\frac{I}{W}\right)^2}{q},\tag{8}$$

The problem is solved for its balanced growth path solution. Recalling the production function Y = BW, the growth rate of output is $g = \frac{I}{W}$. This means that equation 7 can be rewritten as:

$$q = 1 + \theta g \tag{9}$$

In a balanced growth path, the growth rate must be constant, which implies that q must be constant. Therefore equation 8 becomes:

$$q = \frac{B + \frac{1}{2}\theta g^2}{r}$$

Continuing with the description of the model, we turn now to the capital good firms production decisions. Once invented, the physical production of each unit of the specialised capital good requires one unit of capital. So, in each period the monopolistic capital good producer maximises its profits, taking as given the demand curve 6 for its good:

$$\max_{x(t)} \pi(t) = R(t)x(t) - rqx(t),$$

which leads to the markup rule:

$$R = \frac{rq}{\gamma}$$

At time t, in order to enter the market and produce the Ath capital good, a firm must spend upfront an amount given by $P_A A(t)^{\xi}$, where, as mentioned earlier, P_A is the fixed price of one new design in units of foregone output, and i^{ξ} represents an additional cost of patent i in terms of foregone output. Hence, the dynamic zero-profit/free-entry condition is:

$$P_A A(t)^{\xi} = \int_t^{\infty} e^{-r(\tau-t)} \pi(\tau) d\tau,$$

which, assuming no bubbles is equivalent to:

$$\xi g_A = r - \frac{\pi}{P_A A^{\xi}},\tag{10}$$

In a balanced growth path, x is growing at the rate:

$$\frac{\dot{x}}{x} = \frac{\xi A A^{\xi - 1} L\left(\frac{\alpha}{R}\right)^{\frac{1}{1 - \alpha}}}{L A^{\xi} \left(\frac{\alpha}{R}\right)^{\frac{1}{1 - \alpha}}} = \xi g_A$$

Consequently physical capital is growing at the rate:

$$g_k = (1+\xi)g_A,$$

and output is growing at the rate:

$$g_y = \phi g_A + \alpha \xi g_A = (1+\xi)g_A$$

It follows, from equation 3, that total capital W grows at the same rate as output:

$$g_w = \left[\frac{(1+\xi)K + P_A A^{\xi+1}}{W}\right] g_A = (1+\xi)g_A$$

Equation 10 then leads us to the equation that describes the decisions made on the production side:

$$g_y = \frac{1+\xi}{\xi} \left[r - \frac{\pi}{P_A A^\xi} \right],$$

which, recalling and rearranging the expression for profits as:

$$\pi = (R - rq) x = \left(\frac{1 - \gamma}{\gamma}\right) rqLA^{\xi} \left(\frac{\alpha\gamma}{rq}\right)^{\frac{1}{1 - \alpha}},$$

becomes:

$$g = \frac{1+\xi}{\xi} \left[r - \frac{\Omega}{(rq)^{\frac{\alpha}{1-\alpha}}} \right], \qquad \Omega = \frac{\left(\frac{1-\gamma}{\gamma}\right) L\left(\alpha\gamma\right)^{\frac{1}{1-\alpha}}}{P_A} \tag{11}$$

Equation 11 unites the equilibrium balanced growth path pairs (g, r) on the production side of this economy. We call it Technology curve, after Rivera-Batiz and Romer (1991).

3.3 General Equilibrium

The capital accumulation equation 5 tells us that a constant growth rate of W implies that consumption grows at the same rate as output. Which means that, and as labour is constant, the per-capita economic growth rate is given by:

$$g_c = g_y = g_k = g_w = g = (1 + \xi)g_A$$

The general equilibrium solution is obtained by solving the system of the two equations 1 and 11 in the two unknowns, r and g. Recalling equation 9, the system to be solved is:

$$\begin{cases} g = \frac{1}{\sigma}(r-\rho) \\ g = \frac{1+\xi}{\xi} \left[r - \frac{\Omega}{(r+r\theta g)^{\frac{\alpha}{1-\alpha}}} \right] , \qquad r > g > 0, \end{cases}$$

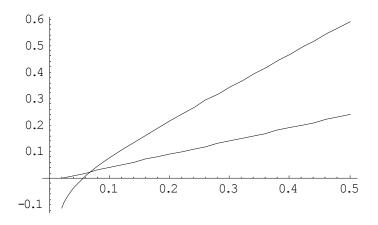


Figure 1:

where $\Omega = \frac{\left(\frac{1-\gamma}{\gamma}\right)L(\alpha\gamma)^{\frac{1}{1-\alpha}}}{P_A}$ and the restriction r > g > 0 is imposed so that present values will be finite, and also so that our solution(s) have positive values for the interest rate and the growth rate.

The Euler equation 1 is linear and positively sloped in the space (r, g). The Technology curve 11 is nonlinear, as shown in the Appendix. Since the nonlinearity of the Technology curve does not allow for the analytical derivation of the equilibrium solution(s), we resort to solving the system through a numerical example. The chosen values for our parameters are:

$$\begin{aligned}
 \sigma &= 2; & \rho = 0.02; & \alpha = 0.4; & \gamma = 0.1; \\
 \xi &= 5; & L = 1; & \theta = 3; & P_A = 5,
 \end{aligned}$$

where the values for α , γ and consequently $\phi = \frac{\alpha}{\gamma}$ and $\xi = \frac{\phi-1}{1-\alpha}$ are the same as those used by Evans et al. (1998) in their numerical example. The values for the preference parameters σ and ρ are in agreement with those found in empirical studies such as Barro and Sala-i-Martin (1995). Population is often chosen to have unity value. And the values for θ and P_A were chosen to give us realistic values for the equilibrium growth rate and interest rate. Although the Technology curve is nonlinear, a unique solution is found. For the adopted parameter values, it is:

$$q = 0.024;$$
 $r = 0.068$

Figure 1, with r on the horizontal axis and g on the vertical axis, helps us visualise this economy's balanced growth path general equilibrium solution.

Proposition 1 A unique solution to this growth model with complementarities and costly investment exists for $\sigma > 1$ and $\Omega^{1-\alpha} > \rho$. **Proof.** Defining two new variables and rewriting our system, we can show that the proposed model does have a unique solution. Our new variables are:

$$Y = \theta g$$
 ; $Z = r(1 + \theta g)$

which allows us to rewrite the system as:

$$\begin{cases} Z^{\beta} = \frac{\lambda}{Y+\mu} \\ Z = \frac{\sigma}{\theta} \left(Y + 1 \right) \left(Y + \eta \right) \end{cases}$$

where $\beta = \frac{\alpha}{1-\alpha}$, $\lambda = \frac{\theta\Omega}{\sigma - \frac{\xi}{1+\xi}}$, $\mu = \frac{\rho\theta}{\sigma - \frac{\xi}{1+\xi}}$, $\eta = \frac{\rho\theta}{\sigma}$. Our restrictions become:

$$Y > 0 \qquad ; \qquad Z > \frac{1}{\theta}Y(Y+1)$$

To ensure that r > g, we impose $\sigma > 1$ so that the Euler equation 1 lies above the 45° line. This implies that λ, μ and η are all positive. Hence the first equation of the rewritten system defines a strictly decreasing curve $Y \mapsto Z(Y)$

from $Z(0) = \left(\frac{\Omega}{\rho}\right)^{\frac{1}{\beta}}$ to $Z(\infty) = 0$, while the second equation defines a strictly increasing curve $Y \mapsto Z(Y)$ from $Z(0) = \rho$ to $Z(\infty) = \infty$. Hence the system has a unique solution in the region Y > 0 iff $\Omega > \rho^{\beta+1}$ (which is equivalent to $\Omega^{1-\alpha} > \rho$). The second restriction is also met because $Z = \frac{\sigma}{\theta} (Y+1) (Y+\eta) > \frac{1}{\theta} Y(Y+1)$.

We conclude that, while the presence of complementarities generally gives rise to multiple equilibria in growth models, in the model developed in this paper, the combination of the assumption of complementarities between capital goods in the production function with the assumption of costly investment in capital generates a single equilibrium.

4 Concluding Remarks

In this paper we have developed a growth model which combines the assumption of complementarities between capital goods in the production function with the assumption of internal costly investment.

Our proposed model builds on the multiple equilibria model by Evans, Honkapohja and Romer (1998) in assuming complementarities between capital goods in the production function. However, we have modified their two-sector structure into a one-sector framework and have replaced their analytically-nonobservable external cost of investment with an analytically-observable internal investment cost function due to Hayashi (1982).

The paper proposes a two-fold contribution to growth theory. Firstly, the introduced assumption of internal costly investment is new to R&D-based growth literature.

Our second proposed contribution is the finding that while growth models with complementarities generally deliver multiple equilibria, in our model the presence of complementarities combined with the existence of internal costs of investment gives rise to a single equilibrium.

Appendix

In order to analyse the shape of the Technology curve 11, and as it is impossible to isolate r on one side of the equation, we rewrite it as F(r, g) = 0 and apply the implicit function theorem, so as to obtain, in the neighbourhood of an interior point of the function, the derivative $\frac{dr}{dg}$ as:

$$\frac{dr}{dg} = -\frac{\frac{dF(r,g)}{dg}}{\frac{dF(r,g)}{dr}}$$

So, we have:

$$F(r,g) = \xi g - (1+\xi)r + (1+\xi)\Omega r^{\frac{-\alpha}{1-\alpha}}(1+\theta g)^{\frac{-\alpha}{1-\alpha}} = 0$$

which leads to:

$$\frac{dr}{dg} = \frac{\xi - \left(\frac{\alpha}{1-\alpha}\right)\theta\left(1+\xi\right)\Omega r^{\frac{-\alpha}{1-\alpha}}\left(1+\theta g\right)^{\frac{-1}{1-\alpha}}}{\left(1+\xi\right) + \left(\frac{\alpha}{1-\alpha}\right)\left(1+\xi\right)\Omega r^{\frac{-1}{1-\alpha}}\left(1+\theta g\right)^{\frac{-\alpha}{1-\alpha}}}$$

Hence, our nonlinear Technology curve is positively sloped when:

$$r^{\frac{-\alpha}{1-\alpha}} \left(1+\theta g\right)^{\frac{-1}{1-\alpha}} < \frac{\xi}{\left(\frac{\alpha}{1-\alpha}\right)\theta\left(1+\xi\right)\Omega}$$

and negatively sloped otherwise.

Replacing the expression for g given by the Euler equation 1 in the Technology curve 11, we obtain the equilibrium expression for r:

$$r\left(\frac{1}{\sigma} - \frac{1+\xi}{\xi}\right) + \frac{\left(\frac{1+\xi}{\xi}\right)\Omega}{\left[\frac{\theta}{\sigma}r^2 + \left(1 - \frac{\theta\rho}{\sigma}\right)r\right]^{\frac{\alpha}{1-\alpha}}} = \frac{\rho}{\sigma}$$

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