# Behaviour-Based Price Discrimination with Elastic Demand*

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**Abstract**

Behaviour-based price discrimination (BBPD) is typically analysed in a framework characterised by inelastic demand. This paper provides a first assessment of the role of demand elasticity on the competitive and welfare effects of BBPD. In contrast to the welfare results derived under the unit demand assumption, the paper shows that BBPD can be welfare enhancing if demand is sufficiently elastic. The demand expansion effect, that is obviously overlooked by the standard framework with unit demand, can play a relevant role. Not only it determines the welfare effects of BBPD but it also explains why BBPD is beneficial to consumers despite it may lead to a slight increase in average prices charged over the two periods.

**JEL**: D43, L13.

**Keywords**: behaviour-based price discrimination, elastic demand, welfare.

## 1 Introduction

The increasing diffusion of the internet as a marketplace and the unprecedented capability of firms to gather and store information on the past shopping behaviour of consumers is enhancing their ability to make use of this information to price differently to their own previous customers and to the

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rivals' consumers. This form of price discrimination, termed behaviour-based price discrimination (BBPD) or price discrimination by purchase history or dynamic pricing, is now widely observed in many markets. Examples of firms that adopt BBPD include supermarkets, web retailers, telecom companies, banks, restaurants and many others.

As this business practice is becoming increasingly prevalent, a good economic understanding of its profit, consumer surplus and welfare implications needs to be founded on a good economic understanding of the market in which it is implemented. Although this type of price discrimination has recently received much attention in economics,[1] the literature has hitherto focused on the assumption that consumers have unit demand. In real markets, however, the consumers' decision does not only involve choosing a firm but also the amount of good(s) purchased. An important issue remains to be explored: What happens if the restrictive assumption of unit demand is relaxed?

The main contribution of this paper is therefore to offer a first assessment of the competitive and welfare effects of BBPD when firms face an elastic demand. The aim is to investigate whether the main results obtained under the unit demand assumption hold or are rather contradicted.

With this goal in mind, we introduce elastic demand into a model of BBPD. Following Fudenberg and Tirole (2000) the paper considers a two-period model with two horizontally differentiated firms competing for consumers with stable exogenous brand preferences across the two periods. These preferences are specified in the Hotelling-style linear market of unit length with firms positioned at the endpoints. Rather than assume that consumers have unit demand, we assume that consumers' demand is elastic. Firms cannot commit to future prices. Because firms have no information about consumers' brand preferences in period 1 they quote a uniform price. In period 2, firms use the consumers' first period purchase history to draw inferences about consumers' preferences and price accordingly.

As mentioned a common assumption adopted by the literature on BBPD is that firms are competing in a unit demand framework *à la Hotelling,* implying that the role of demand elasticity on the competitive effects of BBPD has been mostly overlooked. The assumption may be justified by the challenge posed by elastic demand in a Hotelling framework. Nero (1999) and Rath and Zhao (2001) seem to be the first to tackle the issue. They use quadratic utility preferences to show that a location than price Hotelling game with elastic demand has a unique equilibrium. Both papers emphasise the role of the transport cost to reservation price ratio in determining the

[1] Chen (2005), Fudenberg and Villas-Boas (2007) and Esteves (2009b) present updated literature surveys on BBPD.

optimal location chosen by firms. Anderson and De Palma (2000) introduce constant elasticity of substitution (CES) utility in a spatial framework to analyse issues related to localized and global competition. Gu and Wenzel (2009, 2011) use the same system of preferences to address the optimality of firms' entry in spatial models and the role of information and transparency on the variety supplied by the market, respectively.

In this paper we also adopt a CES utility function to introduce elastic demand in the analysis of competitive BBPD. These preferences allow us to provide a closed form solution to a two periods BBPD model with elastic demand.

The paper has important connections with the literature on BBPD in which firms and consumers interact more than once and firms may be able to learn the consumers' types by observing their past choices and price differently towards them in subsequent periods. In the switching costs approach, consumers initially view the two firms as perfect substitutes; but in the second period they face a switching cost if they change supplier. In this setting, purchase history discloses information about exogenous switching costs (e.g. Chen, 1997 and Taylor, 2003). In the brand preferences approach (e.g. Villas-Boas, 1999, Fudenberg and Tirole, 2000), purchase history discloses information about a consumer's exogenous brand preference for a firm. Although the framework of competition differs in the two approaches their predictions have some common features. First, as in the present model, when price discrimination is permitted, firms offer better deals to the competitor's consumers than to its previous customers. Second, because both firms have symmetric information for price discrimination purposes and the market exhibits best-response asymmetry,[2] industry profits fall with price discrimination.[3,4] Third, when consumers have inelastic demand, there is no welfare benefit when prices fall due to discrimination. The exclusive effect of BBPD is to give rise to a deadweight loss to the society due to the excessive inefficient switching (Fudenberg and Tirole, 2000, Esteves, 2010, Gerigh et al., 2011, 2012). Nonetheless, important differences arise in both approaches when taking into account the effects of poaching on initial prices. While in

[2]In the terminology of Corts (1998) there is best response asymmetry when each firm's strong market is the rival's weak market.

[3]On the Pros and Cons of Price Discrimination, Chen (2005) argues also that firms tend to be worse off being able to recognize consumers and price discriminate. Targeted pricing is also bad for profits in Shaffer and Zhang (1995) and Bester and Petrakis (1996).

[4]There are however some models where firms can benefit from targeted pricing. This conclusion might be obtained when firms are asymmetric (e.g. Shaffer and Zhang, 2000), when firms targetabillity is imperfect and asymmetric (Chen, et al., 2001) and when only one of the two firms can recognize customers and price discriminate (Chen and Zhang, 2009 and Esteves, 2009a).

the brand preferences' approach when BBPD is permitted initial prices are high and then decrease, in the switching costs approach the reverse happens.[5]

Enriching the literature on BBPD to elastic demand is important to investigate in which circumstances the results derived under inelastic demand hold when demand is elastic or are rather contradicted. As expected our framework yields the results derived in the literature when demand is inelastic demand. New results are nonetheless obtained when demand is elastic. When demand is inelastic BBPD reduces average prices in comparison to uniform pricing. The paper shows that this result holds for moderate demand elasticity. In contrast, the paper stresses that average price with BBPD can be above its non discrimination counterpart when elasticity of demand is sufficiently high. Additionally, loyal and poached consumers can face a higher present value of total payment for the two periods of consumption when demand elasticity is high. This reverses the results obtained under inelastic demand.

In comparison to uniform pricing, although average prices can increase with BBPD, we show that in aggregate BBPD always increases overall consumption over the two periods. As a result the paper shows that price discrimination is beneficial to consumers' independently of the effect of BBPD on average prices. Unlike the inelastic case, where price discrimination increases the inefficient switching and reduces welfare, in our context, the demand expansion effect also implies that BBPD can be welfare enhancing: if demand is sufficiently elastic, the higher volume of transactions more than compensates for the increase in transport costs.

The rest of the paper is structured as follows. Section 2 introduces the model. Section 3 sets the benchmark case with no discrimination. Section 4 solves the model when firms practice behaviour based price discrimination. Section 5 discusses the competitive effects of BBPD and Section 6 looks at the welfare effects. Section 7 concludes.

# 2 The model

Two firms, $i = A, B$, produce at zero marginal cost[6] a nondurable good and compete over two periods, $t = 1, 2$. On the demand side, there is a large number of consumers whose mass is normalized to one. In each period a

[5] For other recent papers on BBPD see also Ghose and Huang (2006), Chen and Pearcy (2010), Esteves and Vasconcelos (2012), Caillaud and De Nijs (2011), Ouksel and Eruysal (2011) and Shy and Stenbacka (2011).

[6] The assumption of zero marginal costs can be relaxed without altering the basic nature of the results derived throughout the model.

consumer can either decide to buy the good from firm $A$ or from firm $B$, but not from both. We assume that the two firms are located at the extremes of a unit interval $[0,1]$, and consumers are uniformly distributed along this interval. A consumer situated at $x \in [0,1]$ is at a distance $d_A(x) = x$ from firm $A$ and at distance $d_B(x) = 1-x$ from firm $B$ and $\tau$ is the unit transport cost. Transport cost is linear in distance and does not depend on the quantity purchased. Note that the location of a consumer $x$ represents his relative preference for firm $B$ over $A$ while $\tau > 0$ measures how much a consumer dislikes buying a less preferred brand. A consumer's brand preference $x$ remains fixed for both periods. Following Anderson and De Palma (2000) and Gu and Wenzel (2009, 2011), we write the utility of a consumer buying from firm $i$ as:

$$U_i(x) = v - V(q_i) - \tau d_i(x) - p_i q_i,$$

in which $v$ is the gross utility of consuming the good, $V(q_i)$ is the utility derived by consuming $q_i$ units of the good, $\tau d_i(x)$ is the total transport cost of buying from firm $i$ and $p_i q_i$ is the consumer's expenditure. We shall assume throughout that the reservation value $v$ is high enough such that all consumers purchase in both periods. Assuming preferences display constant elasticity of substitution, type-$x$ consumers' net utility of buying $q_A$ units from firm A at the marginal price $p_A$ can be written as:

$$U_A = v - \frac{1-\sigma}{\sigma} q_A^{-\frac{\sigma}{1-\sigma}} - \tau x - p_A q_A = v - \frac{1}{\sigma} p_A^{\sigma} - \tau x, \tag{1}$$

where $q_i = p_i^{\sigma-1}$, with $\sigma \in (0,1]$. The demand for the differentiated good exhibits a constant demand elasticity of $(1-\sigma)$. A higher value of $(1-\sigma)$ corresponds to more elastic demand. Thus, the limit case of $\sigma \to 1$ (equivalently, $(1-\sigma) \to 0$) corresponds to completely inelastic demand. Thus this demand specification encompasses the standard Hotelling setup with inelastic demand when $\sigma = 1$ and perfect competition as $\sigma \to 0$.

Similarly, if consumer $x$ buys $q_B$ units form firm $B$, the net utility is: $v - \frac{1}{\sigma} p_B^{\sigma} - \tau(1-x)$. These preferences imply that the consumer indifferent between buying from firm A or B is located at:

$$x = \frac{1}{2} + \frac{p_B^{\sigma} - p_A^{\sigma}}{2\tau\sigma}. \tag{2}$$

Firms A and B's demand are given by:

$$D_A = x p_A^{\sigma-1} \quad \text{and} \quad D_B = (1-x)\, p_B^{\sigma-1},$$

while profits are respectively given by $\pi_A = x p_A^{\sigma}$ and $\pi_B = (1-x)\, p_B^{\sigma}$ . In each period firms act simultaneously and non-cooperatively. In the first

period, consumers are *anonymous* and firms quote the same price for all consumers. In the second period, whether or not a consumer bought from the firm in the initial period reveals that consumer's brand preference. Thus, as firms have the required information, they will set different prices to their own previous customers and to the rival's previous customers. If price discrimination is not adopted (for example, if it is forbidden) firms quote again a single price to all consumers. Firms and consumers discount future profits using a common discount factor $\delta \in [0, 1]$.

# 3 No discrimination benchmark

Suppose that for some reason (e.g. regulation, costs of changing prices) firms in the second period can not price discriminate. In that case, the two-period model reduces to two replications of the static equilibrium. To solve for this equilibrium, consider the one period model, and let $p_A$ and $p_B$ denote the prices set by firms A and B, respectively. Firm A solves the following problem:

$$\max_{p_A} \pi_A = \left\{ p_A^\sigma \left( \frac{1}{2} + \frac{p_B^\sigma - p_A^\sigma}{2\tau\sigma} \right) \right\}.$$

From the first-order condition, the best response function is:

$$p_A = \left( \frac{\tau\sigma + p_B^\sigma}{2} \right)^{\frac{1}{\sigma}}.$$

Similarly, firm B's best response function is:

$$p_B = \left( \frac{\tau\sigma + p_A^\sigma}{2} \right)^{\frac{1}{\sigma}}.$$

Solving for the equilibrium, the following proposition can be stated.

**Proposition 1** *In the no discrimination benchmark case equilibrium prices in each period are equal to:*

$$p^{nd} = (\tau\sigma)^{\frac{1}{\sigma}},$$

*and each consumer buys:*

$$q^{nd} = (\tau\sigma)^{\frac{\sigma-1}{\sigma}}.$$

*Thus, each firm's equilibrium profits are*

$$\pi^{nd} = \frac{\tau\sigma}{2}(1 + \delta),$$

*consumer surplus is*

$$CS^{nd} = \left(v - \frac{5}{4}\tau\right)(1+\delta),$$

*and social welfare equals*

$$W^{nd} = \left(v - \frac{5}{4}\tau + \tau\sigma\right)(1+\delta).$$

# 4 Equilibrium analysis

Price discrimination is now feasible. In period 1 because firms cannot recognise customers they set a single first period price, denoted $p_i^1$, $i = A, B$. Consumers' first period choices reveal information about their brand preferences, so firms can set their second period prices accordingly. In the second period, each firm can offer two prices, one to its own past customers, denoted $p_i^o$, and another price to the rival's previous customers, denoted $p_i^r$. To derive the subgame perfect equilibrium, the game is solved using backward induction from the second period.

## 4.1 Second-period pricing

As in Fudenberg and Tirole (2000) the consumers first-period decisions will lead to a cut-off rule, so that first-period sales identify two intervals of consumers, corresponding to each firm's turf. Suppose that at given first-period prices $p_A^1$ and $p_B^1$, there is a cut-off $x_1^*$ such that all consumers with $x < x_1^*$ bought from firm A in period 1. Thus, firm A's turf is the interval $[0, x_1^*]$, while firm B's turf is the remaining $[x_1^*, 1]$.

Look first on firm A's turf (i.e. firm A's strong market and firm B's weak market). Firm A offers price $p_{oA}$, while firm B offers price $p_{rB}$. The marginal consumer, $x_{2A}$ who is indifferent between buying again from firm A and switching to firm B is identified by the following condition:

$$\frac{1}{\sigma}p_{oA}^{\sigma} + \tau x_{2A} = \frac{1}{\sigma}p_{rB}^{\sigma} + \tau(1 - x_{2A}),$$

implying:

$$x_{2A} = \frac{1}{2} + \frac{p_{rB}^{\sigma} - p_{oA}^{\sigma}}{2\tau\sigma}.$$

Each consumer in the market segment $[0, x_{2A}]$ buys $q_{oA} = p_{oA}^{\sigma-1}$ units from firm A in the second period and each consumer in the market segment $[x_{2A}, x_1^*]$ switches to firm B in period 2 and buys $q_{rB} = p_{rB}^{\sigma-1}$ units. Thus,

firm A's demand from retained customers in period 2 is given by $D_{oA} = x_{2A} p_{oA}^{\sigma-1}$ and, similarly, firm B's demand from switching customers is: $D_{rB} = (x_1^* - x_{2A})\, p_{rB}^{\sigma-1}$.

Firm A's second period profit from old customers is:

$$\pi_{oA} = p_{oA} D_{oA} = \left( \frac{1}{2} + \frac{p_{rB}^{\sigma} - p_{oA}^{\sigma}}{2\tau\sigma} \right) p_{oA}^{\sigma},$$

and firm B's second period profit from switching customers is

$$\pi_{rB} = p_{rB} D_{rB} = \left( x_1 - \frac{1}{2} - \frac{p_{rB}^{\sigma} - p_{oA}^{\sigma}}{2\tau\sigma} \right) p_{rB}^{\sigma}.$$

On its turf, firm A chooses $p_{oA}$ to maximise $\pi_{oA}$ for any given $p_{rB}$ yielding the following best response function:

$$p_{oA} = \left( \frac{\tau\sigma + p_{rB}^{\sigma}}{2} \right)^{\frac{1}{\sigma}}.$$

Firm B's best response function on firm A's turf is instead:

$$p_{rB} = \left( \frac{\tau\sigma\,(2x_1 - 1) + p_{oA}^{\sigma}}{2} \right)^{\frac{1}{\sigma}}.$$

Solving for the equilibrium and using an analogous reasoning for firm B's yields the following proposition.

**Proposition 2** *When firms can recognise their old and the rivals' previous customers and price discriminate, second-period equilibrium prices and quantities are:*
*(i) if* $\frac{1}{4} \leq x_1 \leq \frac{3}{4}$ :

$$\begin{aligned} p_{oA} &= \left[ \frac{\tau\sigma\,(2x_1 + 1)}{3} \right]^{\frac{1}{\sigma}} \text{ and } p_{rA} = \left[ \frac{\tau\sigma\,(3 - 4x_1)}{3} \right]^{\frac{1}{\sigma}}, \\ q_{oA} &= \left[ \frac{\tau\sigma\,(2x_1 + 1)}{3} \right]^{\frac{\sigma-1}{\sigma}} \text{ and } q_{rA} = \left[ \frac{\tau\sigma\,(3 - 4x_1)}{3} \right]^{\frac{\sigma-1}{\sigma}}, \end{aligned}$$

$$\begin{aligned} p_{oB} &= \left[ \frac{\tau\sigma\,(3 - 2x_1)}{3} \right]^{\frac{1}{\sigma}} \text{ and } p_{rB} = \left[ \frac{\tau\sigma\,(4x_1 - 1)}{3} \right]^{\frac{1}{\sigma}}, \\ q_{oB} &= \left[ \frac{\tau\sigma\,(3 - 2x_1)}{3} \right]^{\frac{\sigma-1}{\sigma}} \text{ and } q_{rB} = \left[ \frac{\tau\sigma\,(4x_1 - 1)}{3} \right]^{\frac{\sigma-1}{\sigma}}. \end{aligned}$$

*(ii) if* $x_1 \leq \frac{1}{4}$ :

$$\begin{aligned}
p_{oA} &= [\tau\sigma(1-2x_1)]^{\frac{1}{\sigma}} \text{ and } p_{rA} = \left[\frac{\tau\sigma(3-4x_1)}{3}\right]^{\frac{1}{\sigma}}, \\
q_{oA} &= [\tau\sigma(1-2x_1)]^{\frac{\sigma-1}{\sigma}} \text{ and } q_{rA} = \left[\frac{\tau\sigma(3-4x_1)}{3}\right]^{\frac{\sigma-1}{\sigma}}, \\
p_{oB} &= \left[\frac{\tau\sigma(3-2x_1)}{3}\right]^{\frac{1}{\sigma}} \text{ and } p_{rB} = 0, \\
q_{oB} &= \left[\frac{\tau\sigma(3-2x_1)}{3}\right]^{\frac{\sigma-1}{\sigma}} \text{ and } q_{rB} = 0.
\end{aligned}$$

*(iii) if* $x_1 \geq \frac{3}{4}$ :

$$\begin{aligned}
p_{oA} &= \left[\frac{\tau\sigma(2x_1+1)}{3}\right]^{\frac{1}{\sigma}} \text{ and } p_{rA} = 0, \\
q_{oA} &= \left[\frac{\tau\sigma(2x_1+1)}{3}\right]^{\frac{\sigma-1}{\sigma}} \text{ and } q_{rA} = 0, \\
p_{oB} &= [\tau\sigma(2x_1-1)]^{\frac{1}{\sigma}} \text{ and } p_{rB} = \left[\frac{\tau\sigma(4x_1-1)}{3}\right]^{\frac{1}{\sigma}}, \\
q_{oB} &= [\tau\sigma(2x_1-1)]^{\frac{\sigma-1}{\sigma}} \text{ and } q_{rB} = \left[\frac{\tau\sigma(4x_1-1)}{3}\right]^{\frac{\sigma-1}{\sigma}}.
\end{aligned}$$

**Proof.** See the Appendix.

## 4.2 First-period pricing

Consider now the equilibrium first-period pricing and consumption decisions. If firms have no commitment power, their market shares in the first period will affect their second period pricing and profits. Thus, forward looking firms take this interdependence into account when setting their first period prices. As consumers are not myopic they anticipate the firms' second period pricing behaviour. Suppose the first-period prices lead to a cut-off $x_1$ that is in the interior of the interval $[0,1]$. Then the marginal consumer must be indifferent between buying $q_{1A}$ units in the first period at price $p_{1A}$, and buying $q_{rB}$ units next period at the poaching price $p_{rB}$; or buying $q_{1B}$ units in the first period at price $p_{1B}$, and switching to buy $q_{rA}$ units in the second period at the poaching price $p_{rA}$. Hence, at an interior solution:

$$v-\frac{1}{\sigma}p_{1A}^{\sigma}-\tau x_1+\delta\left[v-\frac{1}{\sigma}p_{rB}^{\sigma}-\tau(1-x_1)\right]=v-\frac{1}{\sigma}p_{1B}^{\sigma}-\tau\left(1-x_1\right)+\delta\left[v-\frac{1}{\sigma}p_{rA}^{\sigma}-\tau x_1)\right],$$

yielding:

$$x_1=\frac{1}{2}+\frac{p_{1B}^{\sigma}-p_{1A}^{\sigma}}{2\sigma\tau\left(1-\delta\right)}+\frac{\delta\left(p_{rA}^{\sigma}-p_{rB}^{\sigma}\right)}{2\sigma\tau\left(1-\delta\right)}, \tag{3}$$

in which $p_{rA}$ and $p_{rB}$ are given by the expressions in Proposition 2. Since $p_{rB}=p_{rA}$ when $x_1=\frac{1}{2}$, (3) shows that $x_1=\frac{1}{2}$ exactly when $p_{1A}=p_{1B}$, as expected given the symmetry of the problem. First period profits for firm A and B can be written respectively as:

$$\pi_{1A}=p_{1A}D_{1A}=x_1p_{1A}^{\sigma}, \tag{4}$$

$$\pi_{1B}=p_{1B}D_{1B}=\left(1-x_1\right)p_{1B}^{\sigma} \tag{5}$$

We are now able to characterize the firms' first period problem. Firm A, for example, chooses $p_{1A}$ to maximize its overall profits:

$$\max_{p_{1A}}\pi_A=\pi_{1A}+\delta\pi_{2A},$$

where $\pi_{2A}\left(x_1\left(p_{1A},p_{1B}\right)\right)=x_{2A}p_{oA}^{\sigma}+\left(x_{2B}-x_1\right)p_{rA}^{\sigma}$. Solving the problem allows us to state:

**Proposition 3** *There is a symmetric Subgame Perfect Nash Equilibrium in which:*
*(i) first-period equilibrium price and quantity purchased are respectively given by*

$$p_1=\left[\tau\sigma\left(1+\frac{\delta}{3}\right)\right]^{\frac{1}{\sigma}},$$

$$q_1=\left[\tau\sigma\left(1+\frac{\delta}{3}\right)\right]^{\frac{\sigma-1}{\sigma}}$$

*and both firms share equally the market in period 1, thus* $x_1\left(p_A^1,p_B^1\right)=\frac{1}{2}$*;*
*(ii) second-period equilibrium prices and quantities are:*

$$\begin{aligned} p_o &= \left(\frac{2}{3}\tau\sigma\right)^{\frac{1}{\sigma}}, \quad q_o=\left(\frac{2}{3}\tau\sigma\right)^{\frac{\sigma-1}{\sigma}}, \\ p_r &= \left(\frac{1}{3}\tau\sigma\right)^{\frac{1}{\sigma}}, \quad q_r=\left(\frac{1}{3}\tau\sigma\right)^{\frac{\sigma-1}{\sigma}}. \end{aligned}$$

*and consumers in the intervals* $\left[\frac{1}{3}, \frac{1}{2}\right]$ *and* $\left[\frac{1}{2}, \frac{2}{3}\right]$ *switch from one firm to another in equilibrium.*
*(iii) Each firm's overall profit is equal to*

$$\pi^d = \frac{(8\delta + 9)\,\sigma\tau}{18}.$$

**Proof** See the Appendix.

Note that the properties of CES preferences are such that the level of switching ($S$) is independent of $\sigma$:

$$S = (x_1^* - x_{2A}^*) + (x_{2B}^* - x_1^*) = \frac{1}{3}.$$

Hence, as long as price discrimination is employed, like in the Fudenberg-Tirole model ($\sigma = 1$), one third of total consumers switch to their least favourite firm in the second period regardless of $0 < \sigma \leqslant 1$.

# 5 Competitive effects

The effects of BBPD *vis à vis* non discriminatory prices can now be evaluated. As previously underlined, inelastic demand is captured in our model as a limiting case when $\sigma = 1$. In that case, our results clearly coincide with the received literature (e.g. Fudenberg and Tirole (2000)). Hence, we shall evaluate the impact of price elasticity, $\sigma \in (0, 1)$, on prices, quantities and profits. We consider the effect on prices first.

**Prices** The comparison of the two pricing regimes, uniform and BBPD, allows us to state Proposition 4.

**Proposition 4** *(i) When* $\sigma \in (0, 1]$ *the following relationship between first period, second period and non discriminatory prices holds:*

$$p_r < p_o < p^{nd} \leq p_1,$$

*no matter the elasticity of demand.*
*(ii) As* $\sigma \to 0$ *(perfectly elastic demand):* $p_r = p_o = p^{nd} = p_1 = 0$.
*(iii) Provided that demand is sufficiently elastic and consumers are patient,* $\delta \in (0, 1]$*, the average price paid under BBPD can be higher than the uniform price .*

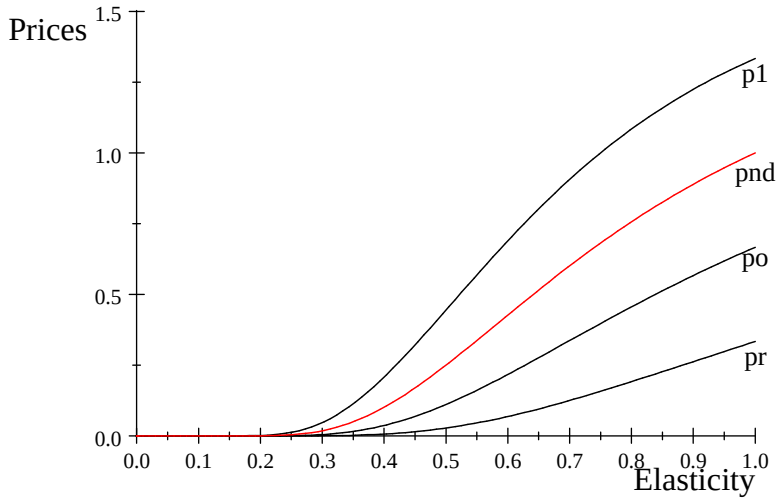


Figure 1: Prices per type of consumers, $\delta = 1, \tau = 1$.

**Proof.** See the Appendix.

The relation between the prices paid by different types of consumers ($p_r < p_o < p^{nd} \leq p_1$) is not affected by the elasticity. As in Fudenberg and Tirole (2000), under BBPD with elastic demand consumers are overcharged in the first period but then strong competition leads to reduced prices in the second period. The reduction is more pronounced for switchers that need to be encouraged to buy their less favourite good. Intuitively, the difference between the prices tends to fade out as demand becomes more elastic. In the extreme case of perfect substitutability between the goods, as expected, prices tend to the marginal cost, i.e. zero. These findings are illustrated in Figure 1. Figure 1 and the ensuing figures are plotted assuming that $\delta = 1$ and $\tau = 1$.

Additional results emerge if we take into account the consumers' present value payment for the two periods of consumption. Without discrimination, each consumer pays a total discounted charge $TPnd = (1+\delta)(\tau\sigma)^{\frac{1}{\sigma}}$ for the two units. With discrimination, loyal consumers in the interval $\left[0, \frac{1}{3}\right]$ and $\left[\frac{2}{3}, 1\right]$ buy from the same firm in each period, the total discounted charge is now equal to $TPo = (\tau\sigma)^{\frac{1}{\sigma}} \left[\left(1+\frac{\delta}{3}\right)^{\frac{1}{\sigma}} + \delta\left(\frac{2}{3}\right)^{\frac{1}{\sigma}}\right]$. While BBPD with inelastic demand ($\sigma = 1$) has no effect on loyal consumers' total payment ($\frac{TPnd}{TPo} = 1$) the same is not true when $0 < \sigma < 1$. With elastic demand it can be shown that $\frac{TPnd}{TPo} < 1$, which means that loyal consumers's present value payment for

the two periods of consumption is higher under BBPD than under uniform pricing.

New results are also obtained for the poached consumers in the interval $\left[\frac{1}{3}, \frac{2}{3}\right]$. These consumers switch from one firm to another and the present value of their payment is equal to $TPr = (\tau\sigma)^{\frac{1}{\sigma}} \left[\left(1+\frac{\delta}{3}\right)^{\frac{1}{\sigma}} + \delta\left(\frac{1}{3}\right)^{\frac{1}{\sigma}}\right]$. With unit demand, this group of consumers pays strictly less moving from no discrimination to BBPD. The same might not be true under elastic demand. In fact, it can be shown that poached consumers can be charged more for the two periods of consumption with BBPD than with uniform pricing ($\frac{TPnd}{TPr} < 1$) when $\sigma \lesssim 0.4$. More precisely, in comparison to uniform pricing, poached consumers are charged more in the two periods with price discrimination if $\sigma$ is such that $(1+\delta) - \left(1+\frac{\delta}{3}\right)^{\frac{1}{\sigma}} + \delta\left(\frac{1}{3}\right)^{\frac{1}{\sigma}} < 0$. When demand elasticity is sufficiently high $\sigma$ (lower than a threshold), the reduction in prices in the second period is not sufficient to compensate for the increase in the first period. As a result of that, total discounted charge increases.

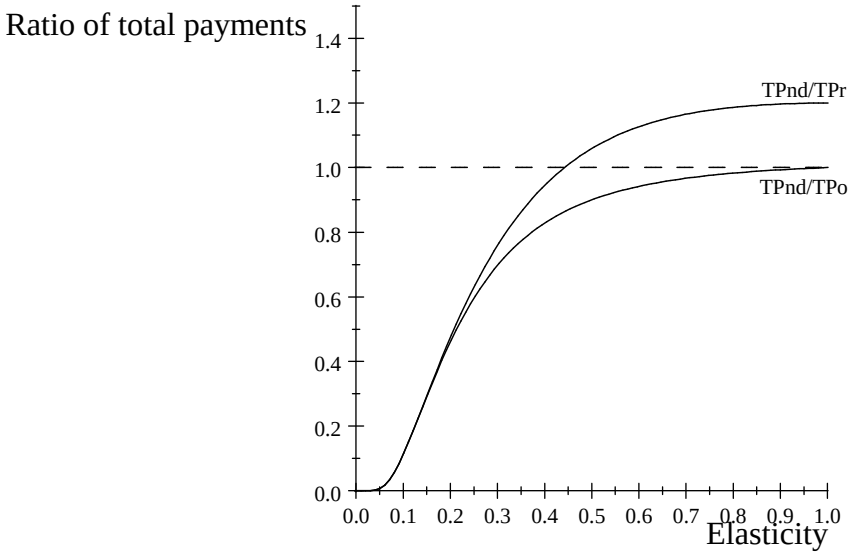


Figure 2: Ratio of total discounted payments

Finally, on average, if demand is inelastic, BBPD leads to lower prices. Interestingly, however, as the elasticity increases this feature of BBPD may no longer hold: the average price paid under BBPD can exceed the uniform price. As mentioned, in comparison to uniform pricing, this tends to be the case when $\sigma$ is such that the reduction in prices in the second period is not sufficient to compensate for the price increase in the first period.

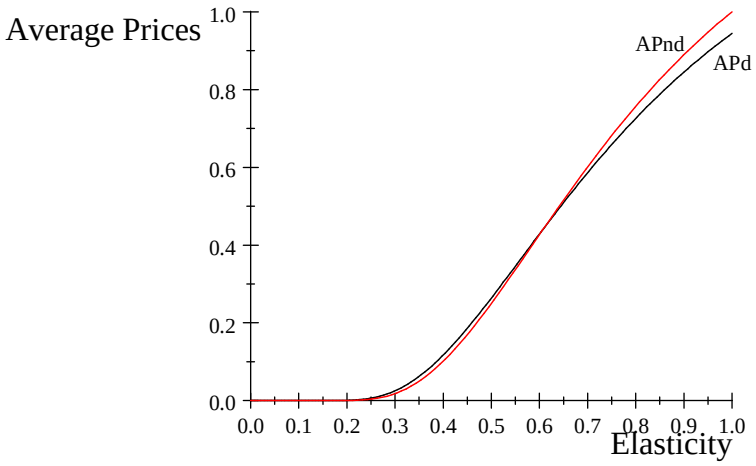


Figure 3: Average prices

**Quantities** Prices clearly affect the quantity demanded by each type of consumer and the overall output supplied. This leads to:

**Proposition 5** *(i) If demand is elastic,* $\sigma \in (0,1)$, *the following relationship between the quantity demanded by each individual consumer in the first period, second period and under no discrimination (each period) holds:*

$$q_1 \leq q^{nd} < q_o < q_r.$$

*(ii) For perfectly inelastic demand,* $\sigma = 1$, *then* $q_1 = q^{nd} = q_o = q_r = 1$.
*(ii) The quantity consumed by any switching consumer over the two periods* $Q^r$ *exceeds the quantity consumed over the two periods by any loyal consumer* $Q^o$*; moreover,* $Q^o$ *exceeds the quantity consumed over the two periods by any consumer under non discrimination* $Q^{nd}$.
*(iii) BBPD increases the aggregate quantity consumed in the market.*

**Proof.** See the Appendix.

In the inelastic benchmark case ($\sigma = 1$), any given consumer demands one unit of the good with and without price discrimination. Elastic demand, instead, implies an inverse relation between price and demand. The consequence is that switching consumers are demanding a higher quantity, both individually and on aggregate through the two periods. Loyal consumers, despite consuming less than switchers, get more of the good than in case

discrimination did not take place. This holds both in the second period and over the two periods: under BBPD the effect of a lower price in the second period leads to a demand increase that more than compensates for the higher price and lower consumption in the first period. In aggregate, this implies that BBPD increases overall consumption over the two periods compared with no discrimination. The results are further illustrated in Figure 4 and 5 that plot quantities as a function of $\sigma$ assuming that $\delta = 1$ and $\tau = 1$.

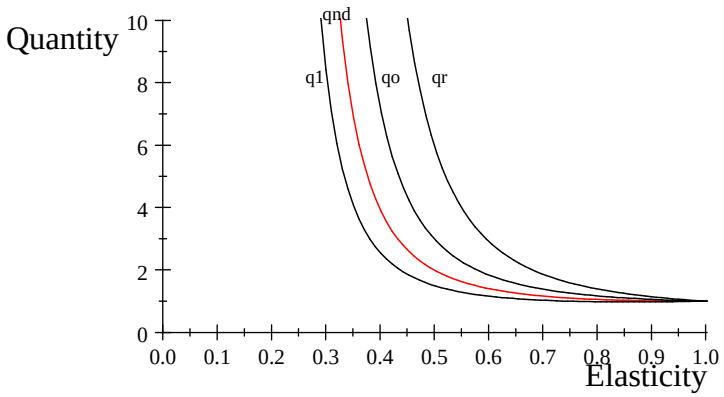


Figure 4: Quantity per consumer per period

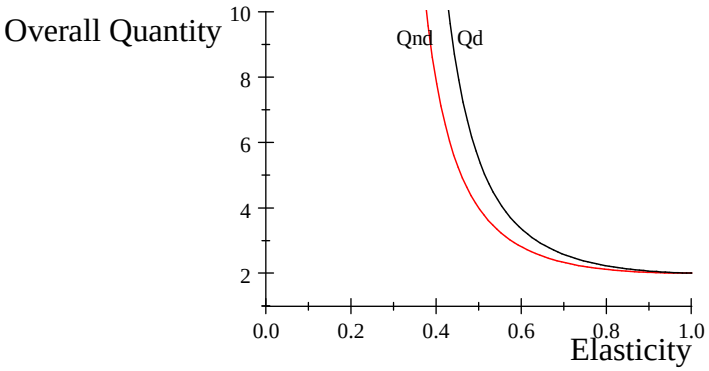


Figure 5: Overall quantity supplied in the two periods

**Profits** Next we look at the profit effects of BBPD with elastic demand. From Proposition 1 follows that under no discrimination industry profits are

equal to $\Pi^{nd} = (1+\delta)\tau\sigma$, while from Proposition 3 under discrimination they are equal to $\Pi^{d} = \frac{(8\delta+9)\tau\sigma}{9}$. The effect of demand elasticity on firms' profits with and without price discrimination is unambiguous. In both pricing regimes, more elastic demand implies lower prices and reduced profit margins and, at the extreme of perfectly elastic demand, industry profits tend to zero. Hence, product market competition is tougher as consumers react more strongly to price changes. As usual, higher product differentiation (higher $\tau$) raises profits.

**Corollary 1.** *BBPD is bad for industry profits. However, the negative impact of BBPD on profits is smaller as demand becomes more elastic.*

The proof of this result is simple, as it stems from the difference in industry profits with no discrimination and BBPD, which is equal to

$$\Pi^{nd} - \Pi^{d} = \frac{1}{9}\sigma\tau\delta > 0.$$

Like in the inelastic case this paper also suggests that a ban on price discrimination acts to promote industry profits. However, the reduction in profits due to price discrimination compared to no discrimination is smaller the more elastic id the demand.

# 6 Welfare analysis

The welfare effects of BBPD with elastic demand can now be investigated. As shown in the previous section, in comparison to uniform pricing, BBPD is always bad for industry profits but it is worse if demand is less elastic.

The aforementioned properties of CES preferences are such that both switching and consumers' surplus are not affected by the elasticity of demand. Consequently as shown in Lemma 1 consumer surplus in our framework is independent of $\sigma$.

**Lemma 1** *With behaviour-based price discrimination overall consumer surplus is:*

$$CS^{d} = v\,(1+\delta) - \frac{5}{4}\tau - \frac{43}{36}\tau\delta$$

*and overall welfare is equal to:*

$$W^{d} = v\,(1+\delta) + \frac{(8\delta+9)}{9}\tau\sigma - \frac{5}{4}\tau - \frac{43}{36}\tau\delta.$$

**Proof.** See the Appendix.

From Proposition 1 we have that consumer surplus with no discrimination equals $CS^{nd} = \left(v - \frac{5}{4}\tau\right)(1+\delta)$. The difference in consumer surplus with uniform pricing and BBPD is:

$$CS^{nd} - CS^{d} = -\frac{1}{18}\tau\delta < 0.$$

Compared with uniform pricing consumer surplus is higher under BBPD. Therefore, the paper shows that the result that BBPD promotes consumer surplus is a robust conclusion, which holds true independently of the elasticity of demand. When demand is inelastic consumer surplus increases with BBPD because prices fall with discrimination. In our framework, no matter the price effect of BBPD, there is also a demand expansion effect that comes into play. This implies that even if the average price increases under BBPD, the demand expansion effect linked to demand elasticity prevails.

Finally, we look at the impact of BBPD with elastic demand on social welfare. From Proposition 1 we have that $W^{nd} = \left(v - \frac{5}{4}\tau + \tau\sigma\right)(1+\delta)$. Therefore, the difference in overall welfare under uniform pricing and BBPD is:

$$W^{nd} - W^{d} = \frac{1}{18}\tau\delta\left(2\sigma - 1\right).$$

**Proposition 6** *BBPD is welfare enhancing if* $\sigma < 0.5$; *it reduces social welfare if* $\sigma > 0.5$. *Finally, if* $\sigma = 0.5$, *BBPD has no effect on overall welfare.*

**Proof.** Straightforward from the expression of the welfare differential $W^{nd} - W^{d}$.■

A relevant contribution of this paper is to highlight that a complete picture of the welfare effects of price discrimination based on purchase history should be drawn under the assumption of elastic demand. As expected, our results confirm that with inelastic demand the exclusive effect of price discrimination is to increase the inefficient switching and to reduce overall welfare (e.g., Fudenberg and Tirole, 2000, Esteves, 2010, Gehrig et al., 2011, 2012).

The extension of the model to elastic demand leads to an important new welfare result. Overall welfare is higher under uniform pricing if demand is sufficiently inelastic $0.5 < \sigma \leq 1$. More interestingly, if elasticity is high enough ($\sigma < 0.5$), the welfare result derived in models with unit demand no

longer applies. Specifically, the welfare analysis puts forward that if demand is sufficiently elastic, price discrimination can actually increase overall welfare in comparison to uniform pricing. The inefficiency created by increased transportation costs (i.e. sub-optimal consumption) is more than compensated by the increase in overall consumption induced by the reduced profit margins that firms can charge.

Our model suggests that regardless of the elasticity of demand BBPD increases price competition in the market in such a way as to transfer wealth from firms to consumers. If consumer surplus is the competition authority's standard, as it is the case in most antitrust jurisdictions, then BBPD tends to benefit consumers and should not be blocked. In contrast, if total welfare is the criterion adopted by the competition authority to evaluate the effects of this business practice, then the policy indication is not as clear cut. If demand elasticity is low a ban on BBPD boosts industry profit and overall welfare at the expense of consumer welfare. If instead, demand is sufficiently elastic (high $\sigma$) in comparison to uniform pricing BBPD promotes not only consumer welfare but also overall welfare. Thus, a ban or deterrence of practices favouring or implementing price discrimination would enhance industry profits at the expense of overall welfare.

# 7 Concluding remarks

This paper constitutes a first assessment of the competitive effects of BBPD when firms face an elastic demand. Enriching the literature on BBPD to elastic demand is important to investigate in which circumstances the results derived under inelastic demand hold when demand is elastic or are rather contradicted.

An important contribution of the paper is to show that in contrast to the welfare results derived under the unit demand assumption, BBPD can be welfare enhancing if demand is sufficiently elastic. The increase in transport costs related to switching, that dominates in the standard inelastic demand framework, is more than compensated by the demand expansion effect implied by the reduced prices and profit margins that firms can charge when demand is elastic. The demand expansion effect, that is obviously overlooked by the standard framework à la Hotelling, can play a very relevant role. Not only it determines the welfare effects of BBPD just discussed but it also explains why BBPD is beneficial to consumers despite it may lead to a slight increase in the average prices charged over the two periods.

A further contribution of this paper is to provide a closed form solution to a model of competitive BBPD with elastic demand. The assumption of

CES preferences is crucial to the goal. Although it is convenient and elegant, the assumption can also be seen as a limitation of our work. Extending the analysis to different or more general preferences is one of the challenges of future research. Finally, as the results were derived in a two period model with consumers' preferences uniformly distributed, further directions for future research might be to address competitive BBPD with elastic demand in an infinite time horizon and other distributions of consumer preferences.

# Appendix

This appendix collects the proofs that were omitted from the text.

**Proof of Proposition 2.** In A's turf, firm A's second period profit from old customers is:

$$\pi_{oA} = \left(\frac{1}{2} + \frac{p_{rB}^{\sigma} - p_{oA}^{\sigma}}{2\tau\sigma}\right) p_{oA}^{\sigma},$$

and firm B's second period profit from switching customers is

$$\pi_{rB} = \left(x_1 - \frac{1}{2} - \frac{p_{rB}^{\sigma} - p_{oA}^{\sigma}}{2\tau\sigma}\right) p_{rB}^{\sigma}.$$

On its turf, firm A chooses $p_{oA}$ to maximise $\pi_{oA}$ for any given $p_{rB}$. The FOC for a maximum yield

$$\frac{1}{2\tau} p_{oA}^{\sigma-1} \left(\tau\sigma - 2p_{oA}^{\sigma} + p_{rB}^{\sigma}\right) = 0$$

As $p_{oA}^{\sigma-1} > 0$ firm A's best response function is

$$p_{oA} = \left(\frac{\tau\sigma + p_{rB}^{\sigma}}{2}\right)^{\frac{1}{\sigma}},$$

On firm A's turf, firm B chooses $p_{rB}$ to maximize $\pi_{rB}$ given $p_{oA}$. The FOC for a maximum yields

$$\frac{1}{2\tau} p_{rB}^{\sigma-1} \left(-\tau\sigma + p_{oA}^{\sigma} - 2p_{rB}^{\sigma} + 2x_1\tau\sigma\right) = 0$$

As $p_{rB}^{\sigma-1} > 0$ firm B's best response function on firm A's turf is instead:

$$p_{rB} = \left(\frac{\tau\sigma\left(2x_1 - 1\right) + p_{oA}^{\sigma}}{2}\right)^{\frac{1}{\sigma}}.$$

From the two best response functions it is straightforward to find that at an interior solution

$$p_{oA} = \left[\frac{\tau\sigma\left(2x_1 + 1\right)}{3}\right]^{\frac{1}{\sigma}} \text{ and } p_{rB} = \left[\frac{\tau\sigma\left(4x_1 - 1\right)}{3}\right]^{\frac{1}{\sigma}}$$

Given that $q_i = p_i^{\sigma-1}$ it follows that

$$q_{oA} = \left[\frac{\tau\sigma\left(2x_1 + 1\right)}{3}\right]^{\frac{\sigma-1}{\sigma}} \text{ and } q_{rB} = \left[\frac{\tau\sigma\left(4x_1 - 1\right)}{3}\right]^{\frac{\sigma-1}{\sigma}}$$

Similar derivations in firm B's turf allow us to obtain $p_{oB}$, $p_{rA}$, $q_{oB}$ and $q_{rA}$.

Finally, note that it is a dominated strategy for each firm to quote a poaching price, $p_{ri}$ below the marginal cost, which in this case is equal to zero. In firm A's turf, from $p_{rB} > 0$ it must be true that $x_1 > \frac{1}{4}$. Otherwise, i.e., when $x_1 \leq \frac{1}{4}$ it follows that $p_{rB} = 0$, and and so firm A's best response in order not to lose the marginal consumer located at $x_1$ is to quote $p_{oA}$ such that $v - \frac{1}{\sigma} p_{oA}^{\sigma} - \tau x_1 = v - \tau (1 - x_1)$. Thus, when $x_1 \leq \frac{1}{4}$ it follows that

$$\begin{aligned} p_{oA}^{\sigma} &= [\sigma\tau (1 - 2x_1)]^{\frac{1}{\sigma}} \text{ and } p_{rB} = 0 \\ q_{oA} &= [\sigma\tau (1 - 2x_1)]^{\frac{\sigma-1}{\sigma}} \text{ and } q_{rB} = 0. \end{aligned}$$

The equilibrium prices in firm B's turf are the same reported above. Similarly it is straightforward to find that if $x_1 \geq \frac{3}{4}$

$$\begin{aligned} p_{oB} &= [\tau\sigma(2x_1 - 1)]^{\frac{1}{\sigma}} \text{ and } p_{rA} = 0 \\ q_{oB} &= [\tau\sigma(2x_1 - 1)]^{\frac{\sigma-1}{\sigma}} \text{ and } q_{rA} = 0 \end{aligned}$$

This completes the proof.■

**Proof of Proposition 3.** With no loss of generality consider the case of firm A. It's overall profit is equal to:

$$\pi_A = x_1 p_{1A}^{\sigma} + \delta [\pi_{oA} + \pi_{rA}]$$

with $\pi_{oA} = x_{2A} p_{oA}^{\sigma}$ and $\pi_{rA} = (x_{2B} - x_1) p_{rA}^{\sigma}$. Given the equilibrium solutions derived in Proposition 2 it follows that:

$$\pi_{oA} = \frac{1}{3}\sigma\tau (2x_1 + 1) \left(\frac{1}{3}x_1 + \frac{1}{6}\right) \text{ and,}$$

$$\pi_{rA} = \frac{1}{3}\sigma\tau (4x_1 - 3) \left(\frac{2}{3}x_1 - \frac{1}{2}\right).$$

Firm A overall profit can be rewritten as:

$$\pi_A = x_1 p_{1A}^{\sigma} + \frac{5\delta\sigma\tau (2x_1^2 - 2x_1 + 1)}{9}$$

with $x_1$ implicitly given by $F(x_1, p_{1B}, p_{1A}) = 0$ such that:

$$x_1 - \frac{1}{2} - \frac{p_{1B}^{\sigma} - p_{1A}^{\sigma}}{2\sigma\tau (1 - \delta)} - \frac{\delta}{2\sigma\tau (1 - \delta)} \left(-\frac{4}{3}\tau\sigma (2x_1 - 1)\right) = 0$$

Firm A's first-order condition is:

$$\frac{\partial \pi_{1A}}{\partial p_{1A}} = \sigma x_1 p_{1A}^{\sigma-1} + \frac{\partial x_1}{\partial p_{1A}} \left( p_{1A}^{\sigma} + \frac{5}{9}\sigma\tau\delta\left(4x_1 - 2\right)\right) = 0$$

with

$$\frac{\partial x_1}{\partial p_{1A}} = -\frac{\frac{\partial F}{\partial p_{1A}}}{\frac{\partial F}{\partial x_1}} = \frac{3}{2}\frac{p_{1A}^{\sigma-1}\left(1-\delta\right)}{\tau\left(\delta-1\right)\left(\delta+3\right)}.$$

Thus from $\frac{\partial \pi_{1A}}{\partial p_{1A}} = 0$ it follows that:

$$p_{1A}^{\sigma-1}\left[\sigma x_1 + \frac{3}{2}\frac{\left(1-\delta\right)}{\tau\left(\delta-1\right)\left(\delta+3\right)}\left( p_{1A}^{\sigma} + \frac{5}{9}\sigma\tau\delta\left(4x_1-2\right)\right)\right] = 0$$

As $p_{1A}^{\sigma-1} > 0$ it must be the case that:

$$\sigma x_1 + \frac{3}{2}\frac{\left(1-\delta\right)}{\tau\left(\delta-1\right)\left(\delta+3\right)}\left( p_{1A}^{\sigma} + \frac{5}{9}\sigma\tau\delta\left(4x_1-2\right)\right) = 0$$

Hence as we are looking for a symmetric equilibrium the FOC evaluated at $x_1 = \frac{1}{2}$ simplifies to:

$$\frac{1}{2}\sigma - \frac{3}{2\tau\left(\delta+3\right)}p_{1A}^{\sigma} = 0,$$

implying that $p_{1A} = p_{1B} = p_1$

$$p_1 = \left[\tau\sigma\left(1+\frac{\delta}{3}\right)\right]^{\frac{1}{\sigma}}.$$

As $q_i = p_i^{\sigma-1}$ we have that $q_1 = \left[\tau\sigma\left(1+\frac{\delta}{3}\right)\right]^{\frac{\sigma-1}{\sigma}}$. This completes the proof of (i). Using the fact of $x_1 = \frac{1}{2}$ is is straightforward to prove part (ii). From the equilibrium prices derived for both periods it is also straightforward to show that $\pi^d = \frac{(8\delta+9)\sigma\tau}{18}$.■

**Proof of Proposition 4.** (i) Consider first $p_1$ and $p^{nd}$. The two prices are identical if and only if $\delta = 0$. If $\delta \in (0,1]$, the argument of $p_1$ dominates the one of $p^{nd}$ as $\tau\sigma\left(1+\delta/3\right) > \tau\sigma$; applying a monotonically increasing transformation to both arguments does not change the relationship so $\forall\sigma \in (0,1]$, $p_1 > p^{nd}$. The difference between $p^{nd}$ and $p^o$ is $\left(1-\left(\frac{2}{3}\right)^{\frac{1}{\sigma}}\right)(\tau\sigma)^{\frac{1}{\sigma}} > 0$, $\forall\sigma \in (0,1]$ implying $p^{nd} > p^o$. A similar argument applies to $p^o$ and $p^r$, whose difference is $\left(\left(\frac{2}{3}\right)^{\frac{1}{\sigma}} - \left(\frac{1}{3}\right)^{\frac{1}{\sigma}}\right)(\tau\sigma)^{\frac{1}{\sigma}} > 0$, $\forall\sigma \in (0,1]$ implying $p^o > p^r$. Finally, it is easy to verify that:

$$\lim_{\sigma\to 0}\left(\frac{1}{3}\tau\sigma\right)^{\frac{1}{\sigma}} = \lim_{\sigma\to 0}\left(\frac{2}{3}\tau\sigma\right)^{\frac{1}{\sigma}} = \lim_{\sigma\to 0}\left(\tau\sigma\right)^{\frac{1}{\sigma}} = \lim_{\sigma\to 0}\left[\tau\sigma\left(1+\frac{\delta}{3}\right)\right]^{\frac{1}{\sigma}} = 0.$$

(ii) As there is no change between the two periods, the average non discriminatory price coincides with $p^{nd}$. The average price paid by consumers under BBPD is:

$$\begin{aligned}\widehat{p}^{d} &= \frac{1}{2}p_1 + \frac{1}{2}\left(\frac{1}{3}p_r + \frac{2}{3}p_o\right) \\ &= \frac{1}{2}\left(\tau\sigma\left(1+\frac{\delta}{3}\right)\right)^{\frac{1}{\sigma}} + \frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{3}\tau\sigma\right)^{\frac{1}{\sigma}} + \frac{2}{3}\left(\frac{2}{3}\tau\sigma\right)^{\frac{1}{\sigma}}\right).\end{aligned}$$

Both $\widehat{p}^{d}$ and $p^{nd}$ are increasing functions of $\sigma$ over the domain. The two prices are clearly identical as $\sigma \to 0$; moreover, $p^{nd} > \widehat{p}^{d}$ for $\sigma = 1$. Provided that $\delta > 0$:

$$\lim_{\sigma\to 0}\frac{p^{nd}}{\widehat{p}^{d}} = \frac{(\tau\sigma)^{\frac{1}{\sigma}}}{\frac{1}{2}\left(\frac{\tau\sigma(\delta+3)}{3}\right)^{\frac{1}{\sigma}} + \frac{1}{6}\left(\frac{1}{3}\tau\sigma\right)^{\frac{1}{\sigma}} + \frac{1}{3}\left(\frac{2}{3}\tau\sigma\right)^{\frac{1}{\sigma}}} = 0^{+},$$

implying that $\widehat{p}^{d} > p^{nd}$ as $\sigma \to 0$. Hence, we can conclude that the two functions intersect for at least one value of $\sigma \in (0,1)$.■

**Proof of Proposition 5.** (i) As demand is inversely related to prices, the results follow from Proposition 4 (i). In particular, as $\sigma \in (0,1)$, the function $X^{\frac{\sigma-1}{\sigma}}$ is decreasing for any value of the argument $X$; hence, for any given value of $\sigma$, the smaller the argument, the larger $X^{\frac{\sigma-1}{\sigma}}$. But then $\tau\sigma\left(1+\frac{\delta}{3}\right) \geq \tau\sigma > \frac{2}{3}\tau\sigma > \frac{1}{3}\tau\sigma$ implies $q_1 \leq q^{nd} < q_o < q_r$. Finally, it can also be verified that if demand is perfectly inelastic:

$$\lim_{\sigma\to 1}\left(\frac{1}{3}\tau\sigma\right)^{\frac{\sigma-1}{\sigma}} = \lim_{\sigma\to 1}\left(\frac{2}{3}\tau\sigma\right)^{\frac{\sigma-1}{\sigma}} = \lim_{\sigma\to 1}(\tau\sigma)^{\frac{\sigma-1}{\sigma}} = \lim_{\sigma\to 1}\left[\tau\sigma\left(1+\frac{\delta}{3}\right)\right]^{\frac{\sigma-1}{\sigma}} = 1.$$

(ii) The quantity consumed over two periods by a given switching consumer is:

$$Q^{r} = \left[\tau\sigma\left(1+\frac{\delta}{3}\right)\right]^{\frac{\sigma-1}{\sigma}} + \left(\frac{1}{3}\tau\sigma\right)^{\frac{\sigma-1}{\sigma}}. \tag{6}$$

The corresponding quantity consumed by a loyal consumer is:

$$Q^{o} = \left[\tau\sigma\left(1+\frac{\delta}{3}\right)\right]^{\frac{\sigma-1}{\sigma}} + \left(\frac{2}{3}\tau\sigma\right)^{\frac{\sigma-1}{\sigma}}, \tag{7}$$

while any given consumer under no discrimination consumes:

$$Q^{nd} = 2\left(\tau\sigma\right)^{\frac{\sigma-1}{\sigma}}. \tag{8}$$

Focus first on (6) and (7). The first term is identical in both but from point (i) we know that $q_o < q_r$, $\sigma \in (0,1)$. Hence, $Q^r > Q^o$. Turning to (7) and (8), we can write the difference of the two as:

$$\Delta Q = Q^o - Q^{nd} = \underbrace{\left[\left(\tau\sigma\left(1+\frac{\delta}{3}\right)\right)^{\frac{\sigma-1}{\sigma}} - \left(\tau\sigma\right)^{\frac{\sigma-1}{\sigma}}\right]}_{A} + \underbrace{\left[\left(\frac{2}{3}\tau\sigma\right)^{\frac{\sigma-1}{\sigma}} - \left(\tau\sigma\right)^{\frac{\sigma-1}{\sigma}}\right]}_{B},$$

from part (i) we know that $A \leq 0$ while $B > 0$. In case $\delta = 0$ then the result is obvious. If, instead, $\delta \in (0,1]$, we consider once again the function $X^{\frac{\sigma-1}{\sigma}}$; as the function is decreasing in $X$ then $|A| - B = \left(\frac{2}{3}\tau\sigma\right)^{\frac{\sigma-1}{\sigma}} - \left[\tau\sigma\left(1+\frac{\delta}{3}\right)\right]^{\frac{\sigma-1}{\sigma}} < 0$ implying $\Delta Q > 0$.

(iii) The result descends immediately from point (ii). As the market is covered under both no discrimination and BBPD and as both switchers and loyal consumers consume over the two period more than any consumer under no discrimination, then surely BBPD increases the overall quantity consumed, or $Q^d = \frac{1}{3}Q^r + \frac{2}{3}Q^o > Q^{nd}$.■

**Proof of Lemma 1.** Overall consumer surplus under BBPD is obtained as:

$$\begin{aligned}
CS^d &= 2\left[\int_0^{\frac{1}{2}}\left(v - \frac{1}{\sigma}p_1^\sigma - \tau x\right)dx\right] \\
&\quad +2\delta\left[\int_0^{\frac{1}{3}}\left(v - \frac{1}{\sigma}p_o^\sigma - \tau x\right)dx + \int_{\frac{1}{3}}^{\frac{1}{2}}\left(v - \frac{1}{\sigma}p_r^\sigma - \tau(1-x)\right)dx\right] \\
&= 2\left[\int_0^{\frac{1}{2}}\left(v - \left(1+\frac{\delta}{3}\right)\tau - \tau x\right)dx\right] \\
&\quad +2\delta\left[\int_0^{\frac{1}{3}}\left(v - \frac{2}{3}\tau - \tau x\right)dx + \int_{\frac{1}{3}}^{\frac{1}{2}}\left(v - \frac{1}{3}\tau - \tau(1-x)\right)dx\right] \\
&= v\left(1+\delta\right) - \frac{5}{4}\tau - \frac{43}{36}\tau\delta
\end{aligned}$$

As industry profits with discrimination are:

$$\Pi^d = \frac{(8\delta + 9)}{9}\tau\sigma$$

total welfare with discrimination is:

$$\begin{aligned} W^d &= \Pi^d + CS^d \\ &= v\,(1+\delta) + \frac{(8\delta+9)}{9}\tau\sigma - \frac{5}{4}\tau - \frac{43}{36}\tau\delta. \blacksquare \end{aligned}$$

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