A test of collusive behavior based on incentives

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Abstract

This paper proposes a novel collusion test based on the analysis of incentives faced by each firm in a colluding coalition. In fact, once collusion is in effect, each colluding firm faces the incentive to secretly deviate from the agreement, since it thereby increases its profits, although the colluding firms' joint profit decreases. Thus, in a colluding coalition each firm has marginal revenues, calculated with Nash conjectures, which are larger than its marginal costs. The collusion test is based on the rejection of the null hypothesis that the firm marginal revenues with Nash conjectures are equal to or less than its marginal costs.

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1. Introduction

The variety and audacity of secret industry cartels does not cease to surprise. Cartels are found in many industries and regions such as the international vitamin makers, the US electric turbine manufacturers, the German cement makers, the French mobile telecommunications operators, just to cite a few.[1] Despite the threat of large fines if the cartels are detected, recent cases attest to collusion's enduring appeal: US Department of Justice antitrust investigations into alleged price fixing practices at the world's leading flat panel display makers and of the $2 trillion municipal bond market, and investigations by the German cartel office and the European Commission into the German and European electricity markets. Importantly, a large number of cartels are discovered only through denunciation by one of the cartel members, which suggests regulators, when unable to observe conduct, do not seem able to determine misconduct by analysis of structure or performance to a standard of certainty that will stand the scrutiny of the courts.

A large body of research has proposed different tests of conduct consistent with collusive behavior (Feuerstein, 2005; Harrington, 2005; Porter, 2005). It is generally accepted that collusive conduct cannot alone be inferred from market performance indicators such as high profitability, high price-cost margins, or high prices (Demsetz, 1973; Scherer and Ross, 1990; Harrington, 2005).[2] The structuralist hypothesis that market structure is, *per se*, the fundamental explanation for firm conduct is unsatisfactory since it implies the linkage is mechanical, and since it fails to consider the potential for reverse causality (Cabral, 2000), the contestable markets hypothesis (Baumol *et al*, 1982), and to reflect the dynamic nature of oligopolistic competition. Approaches that seek to model firm conduct explicitly (Bresnahan,

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[1] *Handelsblatt*, Apr. 14, 2003, "Zement-Bußgelder über 660 Millionen Euro verhängt"; *Financial Times*, Adam Jones, Dec. 1, 2005, "French top mobile groups fined"; *Financial Times*, Song Jung-a, Dec. 12, 2006, "Samsung, LG Philips and Sharp hit by probe"; and *Bloomberg*, William Selway and Martin Braun, Dec. 7, 2006, "U.S. Criminal Probe Rattles $2 Trillion Municipal Bond Market".

[2] There are apparently plenty of other conducts available to firms to restrict competition which do not necessarily translate in increases in prices or reductions in output relative to the initial

1989; Parker and Röller, 1997) require a degree of rationality, information, and foresight that may not be consistent with reality, make strong assumptions about demand and cost functions, and face significant estimation difficulties. See Corts (1999) for an excellent overview of the problems with such models. Finally, dynamic models of strategic interaction (Harrington, 2005), by emphasizing the possibility of retaliatory behavior due to repeated interaction, offer the possibility of inferring variations in firm conduct over time. However, firm conduct may not be time-consistent nor always consistent with rational behavior in the context of a strategic game. In addition, such models assume some degree of market stability over time, which means they offer unsatisfactory predictions in markets where demand fluctuates strongly or markets that face rapid technological change (Harrington, 2005).

Prior work developed models of firm conduct that focused on the difference between prices and marginal costs as the measure of market performance (Stigler, 1964; Cowling and Waterson, 1976; Bresnahan, 1981; Kamien and Schwartz, 1983; Hall, 1988; Parker and Röller, 1997).

This paper proposes a novel collusion test that sidesteps measures of market performance and instead focuses on the incentives firms face in an imperfectly competitive setting. Economic incentives affect agents' conduct and performance, including in aspects as important as life and death. Dickert-Conlin and Chandra (1999) show how US Federal personal income tax incentives contribute to the anticipation of child births from January to December, and Gans and Leigh (2006) show how 1979 inheritance tax changes in Australia contributed to the delay of the death of 50 individuals by a week, equivalent to more than half of the individuals who would be subject to the inheritance tax. Therefore, it is only logical to seek to detect collusion by looking at the incentives individual firms face rather than by attempting to measure, after-the-fact, the effect of collusion on market performance variables. This is the rationale for the tests proposed in this paper.

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equilibrium. For example, prestigious firms such as Intel or Microsoft have in the past successfully

The main characteristic of a collusive outcome is that it is inherently unstable since it is not a perfect Nash equilibrium and each firm has an incentive to deviate from the agreed output or price (Stigler, 1964; Cabral, 2000).[3] That is, for every possible tacit or explicit collusive agreement, each colluding firm's marginal revenue (with Nash conjectures) is larger than its marginal cost, since each firm is not maximizing its individual profit for the benefit of the colluding coalition joint profits. Under competition *a la Cournot* each firm in the colluding coalition has an incentive to increase output in order to increase its profits. Likewise, under competition *a la Bertrand*, each firm in the colluding coalition has an incentive to lower prices below agreed level so as to increase its profits. Further, the incentive to secretly deviate from the collusive agreement is larger the larger the price-cost margins are (Stigler, 1964; Harrington, 2005; Porter, 2005). In fact, the paper shows that a test of collusive conduct can be accomplished by comparing firm marginal revenues (with Nash conjectures) with its marginal costs, rejecting the hypothesis of no collusion if firm marginal revenues are statistically significantly larger than its marginal costs.

To derive the theorems and proofs, I use the standard imperfect competitive market assumptions, weak-concave and downward sloping demand, linear costs, and Nash conjectures. I deviate from standard oligopoly theory only in that I offer a more precise definition of industry market power, and this definition is not necessary for deriving the results. It can be dropped without affecting the proofs.

This paper is organized as follows. Section 2 focuses on the duality of the Cournot and Bertrand models, to motivate the utilization of Nash conjectures in detecting collusive behavior, and as a basis for the future extension of the theorems and proofs to the Bertrand model. Using a Cournot competition model, Section 3 derives the theorems and proofs that

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threatened suppliers or clients of a competitor, to the detriment of the competitor's business.

[3] In the late 1990s the Economist pointed out that the cheating that the OPEC cartel countries did on their allocated production quotas were the cause for the low oil prices, although there was a modicum of collusion (the International Energy Agency estimated a compliance of 75%). According to the Economist, quoting Vahan Zanoyan of the Washington-based Petroleum Finance Company, "When prices are strong, the temptation to be the first cheat is impossible to resist" (see "Lying low", The Economist, July 2nd 1998; see also "Still kicking?", The Economist, March 25th 1999").

show that a test of collusion can be equated with a test of the profit maximization condition with Nash conjectures. Section 4 specifies the one-tailed test hypothesis consistent with the theorems established in Section 3. Section 5 identifies issues for further research. Section 6 concludes.

2. Cournot and Bertrand Duality and importance of Nash conjectures

As is well known, Cournot and Bertrand oligopoly models differ in what is thought to be the decision variable of the firm, quantity and price, respectively. As a consequence, Cournot and Bertrand equilibria offer radically different predictions for the outcomes of imperfectly competitive markets. If the number of firms is larger than one, Cournot competition is always more “monopolistic” than Bertrand competition, and the latter always achieves equilibria where prices equal marginal costs (Singh and Vives, 1984). A large body of literature has sought reconcile the predictions of these models. This is accomplished through, for example, the introduction of product differentiation, capacity constraints, or a temporal dimension in the Bertrand model, or consistent conjectural variations and barriers to entry in the Cournot model.

Less well emphasized is the fact that, apart from the decision variable, both model frameworks are identical. Both are based on an identical set of beliefs about other firms reactions, assumed not to change when the firm changes its own decision variable, an assumption which I designate here by Nash conjectures but Kamien and Schwartz (1983) designate by zero conjectural variations. As a result of the assumption of Nash conjectures, Cournot and Bertrand equilibria are perfect commitment Nash equilibria, as no firm has the incentive to unilaterally change its decision variable (Singh and Vives, 1984; Fudenberg and Tirole, 1986). Arising from this set of beliefs, Grossman (1981) argues that under Bertrand firms cannot make binding price contracts, while under Cournot, output contracts are always binding. Furthermore, under linear demand and costs, Cournot and Bertrand substitute and complementary goods equilibria have been shown to be dual (Singh and Vives, 1984).

Prior literature (Chamberlin, 1933; Grossman, 1981; Bresnahan, 1981; Kamien and Schwartz, 1983) has criticized Cournot Nash conjectures as a not appropriate model of firm conduct. Among the arguments offered, contrary to the assumptions underlying Cournot Nash conjectures, other firms do react to changes in the firm's output decisions; beliefs underlying Cournot Nash conjectures are logically inconsistent, as firms react to changes in other firms' outputs while assuming that its rivals do not do so; Cournot Nash conjectures are inconsistent with tacit collusion (Chamberlin, 1933); it is argued that price, not output, is the relevant strategic variable in imperfectly competitive markets with homogeneous goods; and, finally, empirical evidence for a variety of industry is inconsistent with Cournot Nash conjectures (Kamien and Schwartz, 1983). Many other authors have also pointed out that the non-cooperative one-shot game reflects reality poorly. These criticisms have been often misinterpreted as criticisms of the Cournot Nash equilibrium, rather than of the set of beliefs, i.e., the underlying Cournot Nash conjectures. Moreover, as was seen above, most of the above arguments apply also to Bertrand Nash conjectures.

Nonetheless, there is a strong case for using Cournot and Bertrand Nash conjectures for solving the firm profit maximization problem (Daughety, 1985; Corts, 1999). Foremost, the power of this set of beliefs in describing firm interactions in one-shot "prisoner dilemma" type games or when output and price decisions by firms are not perfectly observable by competitors. In addition, Nash conjectures ensure that the Cournot and Bertrand Nash equilibria are perfect commitment equilibria (Fudenberg and Tirole, 1986). If other firms, for some reason such as irrationality or imperfect information, deviate from their optimal response, a profit maximizing firm can use Nash conjectures to derive the optimal off-path equilibrium response, which results in the so-called reaction function.

Finally, this paper argues that Nash conjectures are particularly appropriate for inferring tacit and explicit collusive behavior. Given that collusion is taking place, if a colluding firm violates the collusive agreement, it does so since it assumes that its rivals cannot react or since it assumes its rivals will not react, for example, because the rivals do not observe that the firm is violating the collusive agreement. Alternatively, even if output decisions are observable by rivals and the colluding coalition truly enforces discipline in the case of violations to the collusive agreement by a firm, the unconstrained incentive to deviate that each firm faces is still calculated as if the rivals could not observe and react to the firm's output changes, i.e., as if each colluding firm were operating under Nash conjectures. Thus, Cournot and Bertrand Nash conjectures, even if not representative of actual firm practice (Kamien and Schwartz, 1983), provide an important reference point as to the optimal self-interested and non-cooperative conduct of firms, if left to their own devices.

DEFINITION 1. *In an imperfectly competitive industry, define "industry market power" as the extent to which market price (output) exceeds (falls short of) the non-cooperative Nash equilibrium price (output)[4], where maximum market power is consistent with monopoly profit maximization, and minimum market power is consistent with non-cooperative Nash equilibrium.*

Market power is traditionally defined as the extent to which prices differ from marginal costs, for example through measures such as the Lerner index (Motta, 2004; Scherer and Ross, 1990; Tirole, 1988). Under competition *a la Bertrand*, Definition 1 coincides with the traditional literature definition, as Nash equilibrium is such that price equals marginal cost. However, under competition *a la Cournot*, Nash equilibrium price is larger than marginal cost, and output is smaller than perfect competition output (Singh and Vives, 1984). In offering this alternative definition, I build on prior empirical work that indirectly estimate the degree of collusion by the extent observed price-cost margins differ from those under the Cournot Nash equilibrium (Parker and Röller, 1997). See also Corts (1999) for an overview of similar approaches in the empirical literature.

The motivation for introducing a novel definition of market power is that the reference perfect Nash equilibrium consistent with "optimal" oligopoly performance will differ depending

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[4] The non-cooperative Nash equilibrium differs depending on whether competition is *a la Cournot* or *a la Bertrand*.

on what is considered as the correct strategic variable and underlying model (Cournot or Bertrand). Furthermore, with Definition 1, the measure of industry of market power compares the same decision variable, industry output with non-cooperative Nash equilibrium output, and market price with non-cooperative Nash equilibrium price, consistent with the strategic variable of the underlying Cournot or Bertrand model, rather than always compare price with marginal cost. Finally, given the dual way in which market power is defined, it is always possible to measure industry market power, even when firm costs are asymmetric.

DEFINITION 2. *Collusion occurs when firms cooperate (explicitly or tacitly) to increase industry market power relative to the non-cooperative Nash equilibrium.*

Under this definition, collusion also comprehends the cases where tacit cooperation occurs, although tacit collusion is not unlawful in most countries. Hereafter, I focus on the Cournot model with n firms in equilibrium.

3. Model and Theorems

Using a Cournot competition model, assume homogenous output and an oligopolistic industry structure with n firms in equilibrium, each with constant and identical marginal costs, then:

$$\pi_T(Q_T) = \sum_{i=1}^{n} \pi_i(q_i) = p(Q_T)\sum_{i=1}^{n} q_i - \sum_{i=1}^{n} CT_i(q_i) = p(Q_T)\cdot Q_T - \sum_{i=1}^{n} CT_i(q_i) \qquad (1)$$

where π_T are the industry's total profits, π_i are firm i's profits, q_i is firm i's output, Q_T is total industry output, $p(Q_T)$ is market demand function, and CT_i is firm i's total cost of production of output q_i. The industry's total costs, CT_T is given by:

$$CT_T(q_1, q_2, ..., q_n) = \sum_{i=1}^{n} mc * q_i = mc * Q_T \qquad (2)$$

Each firm's firm profit maximizing condition is given by:

$$\pi_i'(q_i) = 0 \Leftrightarrow p(Q_T) + p'(Q_T)\cdot\left(1 + \frac{dQ_{-i}}{dq_i}\right)q_i = mc \qquad (3)$$

where Q_{-i} is the sum of the outputs of all other firms in the industry, $\gamma = \frac{dQ_{-i}}{dq_i}$ is the conjectural variation which reflects the expectation of firm *i* about how other firms react in terms of their (joint) output level, Q_{-i}, to a change in firm *i*'s output, q_i.

In a non-cooperative equilibrium each firm chooses output to maximize its own profits given other firms optimal output decisions and has Cournot Nash conjectures ($\gamma = \frac{dQ_{-i}}{dq_i}$=0), i.e., assumes absence of reaction by other firms. Thus, expression (3) reduces to the Cournot equilibrium:

$$mr_i^o = mc \tag{4}$$

where mr_i^o, q_i^0 and Q_T^0 are the firm marginal revenues, firm output, and the industry output in a Cournot equilibrium, respectively.

Hereafter, following the arguments presented in Section 2, I distinguish between the firm marginal revenue calculated with Nash conjectures, mr_i, and firm marginal revenue at the Cournot Nash equilibrium, mr_i^o. mr_i coincides with (4) at the Cournot Nash equilibrium, but differs from (4) for all other possible oligopoly outcomes.

ASSUMPTION 1. *For any Q_T>0 such that $p(Q_T)$>0, assume that:*

(i) *Demand is downward sloping, $p'(Q_T)<0$*

(ii) *Demand is weakly concave,* $p''(Q_T) \leq 0$

It follows from Assumption 1 that the industry profit function is strictly concave, i.e.,

$$\pi_T''(Q_T) = p''(Q_T) \cdot Q_T + 2 \cdot p'(Q_T) < 0 \tag{5}$$

and that firm *i*'s profit function with Cournot Nash conjectures, $\gamma = \frac{dQ_{-i}}{dq_i}$=0, is also strictly concave:

$$\pi_i''(q_i) = p''(Q_T) \cdot q_i + 2 \cdot p'(Q_T) < 0, \quad \forall i \in 1,...,n \tag{6}$$

Industry profits under collusion are given by:

$$\pi_T(Q_T^*) = \sum_{i=1}^{n} \pi_i(q_i^*) \tag{7}$$

where maximum industry profits under collusion are always equal to or less than monopoly profits, and firm and industry outcomes under collusion are hereafter identified with an asterisk superscript. Thus, the following condition can be additionally imposed:

ASSUMPTION 2. *For any collusive outcome with industry output* Q_T^* *>0, assume that:*

$$\pi_T'(Q_T^*) \leq 0 \tag{8}$$

Since, from Assumption 1, industry profits are strictly concave, Assumption 2 implies that, under collusion, industry output is equal to or larger than monopoly output, since it would be sub-optimal for the colluding coalition to reduce output below monopoly output levels, where joint industry profitability is increasing in output.

LEMMA 1. *A profit maximizing firm colludes if collusion is a profit enhancing activity.*

Proof. I want to show that (i) collusive behavior by a firm implies an increase in the firm profits relative to the *ex-ante* case where the firm does not collude. The transpose is that (ii) if collusion results in profits that are equal to or smaller than the non-collusive case, then the firm does not collude.

(i) Proof by transposition and *reduction ad absurdum*. Suppose that the firm colludes and that profits under collusion are equal to or smaller than the *ex-ante* non-cooperative profits. Consider first the case of explicit collusion. It is costly to explicitly coordinate actions with other firms rather than act independently. Moreover, there is a, however small, risk of punishment by authorities in the case of discovery of explicit coordination, which has a negative impact on expected profits. In addition, there is also the risk of secret violations of the collusive agreement by competitors, also with a negative impact on expected profits.

Therefore, the firm could reduce costs and increase expected profits by not engaging in explicit collusion, *ceteris paribus*.

A similar reasoning can be applied to the case of tacit coordination. Although tacit coordination is not subject to punishment with negative expected impact on profits, there are no *a priori* reasons why tacit coordination should be preferred to no coordination, unless there are some incentives to do so. Tacit coordination requires at least as much firm effort and resources as non-cooperative profit maximization. Thus, tacit coordination is not less costly than non-cooperative profit maximization. Moreover, there is also the risk of cheating by competitors with a negative impact on expected profits. Therefore, the firm could increase profits by not engaging in tacit collusion.

Since the firm is profit maximizing, if collusion is not a profitable activity then it follows that the firm will not collude, contradicting our initial hypothesis.

Q.E.D.

Note that if collusion is in effect prior to the participation of the firm, there would not be an incentive for the firm to join in the collusion effort *ex-post*, as the *ex-ante* non-colluding firm profits would be larger than under collusion. Alternatively, the colluding coalition might engage in punishing behavior to induce the non-colluding firm to join in.

AXIOM 1. *Collusion is profitable if and only if all low-cost firms collude and if low-cost firms have a sufficiently large combined share of the market.*

A formal proof of this Axiom falls outside the scope of this paper, and therefore it is offered here as an Axiom rather than as a Theorem. The explanation of the Axiom is laid out below, which builds on merger literature and on an example.

Obviously, more than one firm must collude in order for collusion to be profitable.[5] The issue is whether it is possible to characterize the minimum colluding coalition size required in order for collusion to be profitable. Since there is a continuum of possible colluding outcomes, it is not possible to determine the minimum colluding coalition size. However, since perfect collusion achieves, at best, a performance (i.e., profits) identical to that achieved through the merger of the firms, the minimum profitable merger size is the lower bound for the minimal coalition size. Thus, it is useful to build on the horizontal merger literature that seeks to determine the conditions under which mergers among firms are profitable (Salant *et al*, 1983; Perry and Porter, 1985; Farrell and Shapiro, 1990; Cheung, 1992; Faulí-Oller, 1997, 2002).

Salant *et al* (1983) seminal paper showed that following a merger among symmetric firms with linear costs and homogeneous goods, the parties to the merger (insiders) would have the incentive to reduce output, whereas the parties external to the merger would increase output. Moreover, industry output would decrease and price would increase. Building on this result, Cheung (1992) shows that the mergers are always unprofitable if they involve less than 50% of the firms in the industry, but may be profitable if they involve more than that percentage of firms, using strictly concave industry profit function such as that defined in equation (5) of this paper. Faulí-Oller (1997) generalizes the model to show that the minimal market share required to ensure that the merger is profitable is increasing in the degree of concavity of demand and in the *ex-ante* number of firms in the industry.

Thus, the horizontal merger literature indicates that the condition for successful collusion is that the number of firms in the industry is not too large, and that a large percentage of firms participate in the collusive agreement, i.e., collusion would be quite difficult to achieve for a symmetric oligopoly. This result is somewhat paradoxical as it

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[5] If only firm *i* colludes, it follows that firm *i* profits under collusion are lower than its profits under Cournot equilibrium since firm i's profit function is strictly concave and, per definition, the Cournot equilibrium level is profit maximizing for firm i, given all other firms optimal responses. Thus, firm i could increase profits by not engaging in collusion, thus violating Lemma 1.

suggests that both mergers but also tacit and explicit collusion would tend to occur very scarcely, a result contrary to the anecdotal and empirical evidence available.

Three main approaches suggest ways in which there can be profitable mergers with smaller subsets of firms, namely cost convexity (Perry and Porter, 1985), Stackelberg leadership (Daughety, 1990), and cost asymmetry (Faulí-Oller, 2002). Still, Heywood and McGinty (2007) point out that there is a second merger paradox since, even when the merger is profitable, firms quite often face an incentive not to merge, as firms that do not merge often gain more from the merger than the firms that merge. A similar principle applies to collusion, why collude if the firm can "free-ride" and benefit from the collusive efforts of others, without incurring the costs and risks associated with collusive behavior.

Cost asymmetries provide the best line of explanation for these paradoxes. Faulí-Oller (2002) shows that a merger between a low- and a high-cost firm can be profitable, as the merged firm switches production to its low cost facilities.[6] This result is not, in itself, helpful for collusion analysis since, under collusion, it is not feasible to switch production among firms if there are no side-payments. Instead, anecdotal evidence of explicit collusion cases suggests that colluding coalitions nearly always involve all the large firms in a market, i.e., the firms with large market share. In a Cournot setting, firms with large market shares have low marginal costs. Thus, collusion will only be profitable if a sufficiently large coalition of firms with low marginal costs, and as a result, a high combined share of the market, participate in the collusive effort.

As an example consider the case with linear demand given by

$$P(Q) = A - B * Q \qquad (9)$$

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[6] Faulí-Oller (2002, p.83) incorrectly argues that merger among large firms is always not profitable. This is probably a poorly worded statement, given that his own prior work (1997) characterized conditions under which mergers could be profitable even without cost efficiency effects. Nonetheless, both of his contributions to merger theory have not yet, in my view, gained their well deserved recognition.

and asymmetric linear costs, mc_{Low} and mc_{High}. Assume there are n_{Low} firms with low marginal costs, and n_{High} firms with high marginal costs. Then, it can be shown that the Cournot Nash equilibrium is given by:

$$q_{High} = \frac{A-(n_{Low}+1)\times mc_{High} + n_{Low}\times mc_{Low}}{(n_{Low}+n_{High}+1)\times B} \tag{10}$$

$$q_{High} = \frac{A-(n_{High}+1)\times mc_{Low} + n_{High}\times mc_{High}}{(n_{Low}+n_{High}+1)\times B} \tag{11}$$

where q_{High} and q_{Low} are the output of the high- and low-cost firms, respectively. If the cost asymmetries are sufficiently large, then the low cost firms can have a combined very large market share of the market, and it can be profitable for them to collude even if the high cost fringe firms do not collude. As a result of collusion among large firms, their combined market share falls significantly, while the market share and profits of the fringe (high cost) firms rise substantially. However, if a low cost firm does not collude, then it is not profitable for the remaining low cost firms to collude, a result similar to that predicted by Salant *et al* (1983), regardless of whether or not the high cost firm colludes. Thus, collusion must involve all large (low cost) firms.

For example, if the above model is solved with the following parameter values: mc_{Low} = 2, mc_{High} =20, n_{Low} = 5, and n_{High} =1, A=200, B=1, then the combined market share of the large firms under the Cournot Nash equilibrium is 92%. Each large (low cost) firm market share is 16.4% and the fringe (high cost) firm has a 7.6% market share. If all 5 low cost firms collude perfectly, then it is profitable to collude, but combined market share of colluding firms falls to 57%. However, if one of the large firms does not collude, then collusion is no longer profitable as the combined market share of colluding firms falls to 37.5%.

This Axiom addresses both merger paradoxes and is consistent with the prior literature. Collusion is profitable if all large firms participate, therefore there is an incentive for large firms to participate. On the other hand, no large (low-cost) firm can free-ride on the collusive agreement, since it suffices that there is a large free-rider firm to destroy any collusive effort. If a large firm free-rides on the collusive effort, by maximizing profits under Cournot Nash conjectures, it makes the collusive effort unprofitable for the colluding firms, and therefore the other large firms would have no incentive to collude. Collusion is an all or none proposition for large firms.

Finally, Axiom 1 suggests a key difference between the collusion and merger literature. Collusive effort aims at involving a large combined market share of the firms in the market. On the other hand, as partly argued by Faulí-Oller (2002), merger effort aims at achieving cost efficiencies, and thus only occurs between large (low-cost) and small (high-cost) firms. This happens because only mergers that aim for cost efficiencies are both profitable and feasible in the current regulatory environment. Mergers between two large firms are not profitable since, typically, they do not achieve the minimum market share threshold necessary to ensure that the merger is profitable *a la* Salant *et al* (1983). Mergers between all large firms, while profitable, are not socially accepted nor feasible in the current regulatory context.

THEOREM 1. *Collusion occurs if and only if industry profits are above the level of the non-cooperative Nash equilibrium.*

Proof. The proof is again for competition *a la Cournot*. I want to show that (i) collusion implies that industry profits are above the non-cooperative Cournot equilibrium level; and (ii) the finding of equilibria with industry profits above Cournot equilibrium levels implies that there is collusion.

(i) Proof by *reductio ad absurdum*. From Axiom 1, assume that at least two firms collude by deviating from the Cournot equilibrium output level (represented with the superscript "^{0}") and that industry profits in the new equilibrium (superscript "*") are less than or equal to the non-cooperative equilibrium level. From Lemma 1, it follows that a profit maximizing firm colludes because collusion is a profit enhancing activity. Then,

$$\exists i, j \in 1,...,n \qquad \pi_i(q_i^*) > \pi_i(q_i^0) \wedge \pi_j(q_j^*) > \pi_j(q_j^0) \tag{12}$$

that is, at least two firms have profits higher than the non-cooperative level. From this result and the initial hypothesis that industry profits are less than or equal to non-cooperative Cournot profits, it follows that the sum of all other n-2 firm profits must be lower than their Cournot equilibrium level, and if so then

$$\exists k \neq i, j \in 1,...,n \qquad \pi_k(q_k^*) < \pi_k(q_k^0) \tag{13}$$

Definition 1 and 2 indicate that collusion implies a decrease in industry output and an increase in market price, i.e.,

$$p(Q_T^*) > p(Q_T^0) \tag{14}$$

then from (13) and (14) it follows that

$$q_k^* < q_k^0 \tag{15}$$

Since from Assumption 1, firm profits are strictly concave, and non-cooperative Nash equilibrium profits are maximized with:

$$\pi_k'(q_k^0) = p'(Q_T^0) \cdot q_k^0 + p(Q_T^0) - mc = 0 \tag{16}$$

it follows that

$$\pi_k'(q_k^*) > 0 \tag{17}$$

Thus, firm *k* could increase profits by increasing output, so q_k^* is not a non-cooperative Nash equilibrium for firm *k*. Furthermore, a counter example shows that since:

$$q_k^0 - q_k^* \leq Q_T^0 - Q_T^* \tag{18}$$

then firm *k* could achieve profits higher than the non-cooperative level by increasing output to q_k^0, contradicting (13) and implying that the industry profits under collusion cannot be equal to or less than the non-cooperative Cournot equilibrium level.

Q.E.D.

(ii) Proof by *reductio ad absurdum*. Suppose there is a sustainable equilibrium (identified with the superscript "^{e}") where industry profits are above the non-cooperative Cournot equilibrium and there is no collusion, i.e., firms in the industry do not cooperate explicitly or tacitly. Since from Assumption 1 it follows that the industry profit function is strictly concave then $\pi_T'(Q_T) \leq 0, \quad \forall Q_T \geq Q_M^*$, where Q_M^* is optimal monopoly output, and:

$$\pi_T(Q_T^e) > \pi_T(Q_T^0) \Rightarrow Q_T^e < Q_T^0 \Rightarrow p(Q_T^e) > p(Q_T^0) \tag{19}$$

since $p'(Q_T) < 0$. Then, either (a) all firms have profits above the non-cooperative level or (b) at least one firm has profits below the non-cooperative level. If (a) applies, then the non-cooperative Cournot equilibrium cannot be a Nash equilibrium because there is another non-cooperative equilibrium, which improves profits for all firms in the industry; if (b) applies then for the firms that have profits below the non-cooperative level it follows that:

$$\pi_i(q_i^e) < \pi_i(q_i^0) \Leftrightarrow \left(p(Q_T^e) - mc\right) \cdot q_i^e < \left(p(Q_T^0) - mc\right) \cdot q_i^0 \tag{20}$$

from (19) it follows that

$$\left(p(Q_T^e) - mc\right) > \left(p(Q_T^0) - mc\right) \tag{21}$$

thus, from (20) and (21) for firm *i* to have profits below the non-cooperative level it must be that

$$q_i^e < q_i^0 \tag{22}$$

but then a similar reasoning to part (i) of the proof can be used to show that firm *i* could achieve profits at least as high as the non-cooperative level by increasing output to the Cournot equilibrium level, contradicting our initial hypothesis (b). Thus, the outcome with higher industry profits would not be a sustainable equilibrium, contrary to our initial hypothesis.

Q.E.D.

COROLLARY 1. *If industry profits are larger than the non-cooperative Nash equilibrium industry profits, then the profits of any subset of the firms in the industry are larger than the profits of the same subset of firms under a non-cooperative Nash equilibrium.*

Proof. Both colluding and non-colluding firms benefit from collusion. Theorem 1 shows that if industry profits are larger than the non-cooperative Nash equilibrium level then there is collusion. Lemma 1 shows that firms collude if and only if collusion is profitable. Thus, the profits of any subset of colluding firms is larger than the profits of the same subset of colluding firms under the non-cooperative Nash equilibrium.

Moreover, Salant *et al* (1983), Heywood and McGinty (2007), and others show that firms external to mergers benefit from a merger, often more so than firms that merge. From Assumption 1, it follows that the industry profit function is strictly concave. Assumption 2 additionally means that higher industry profits can only be achieved through higher price and lower output level. As a result, each and every non-colluding profit maximizing firm, experiences a shift to the right of its residual demand function, and achieves higher profits than under the non-cooperative Nash equilibrium. Thus, the joint profit of any subset of non-colluding firms is also larger than the profits of the same subset of firms under the non-cooperative Nash equilibrium.

Q.E.D.

This result is relevant if it is only possible to obtain firm data for a subset of the industry. For example, it maybe the case that it is only possible to obtain data for the largest firms in the industry. Axiom 1, Theorem 1, and Corollary 1 indicate that data on the largest firms would suffice to investigate whether the industry has or not collusive behavior.

THEOREM 2. *Industry profits are above non-cooperative Nash equilibrium levels if and only if marginal revenue with Nash conjectures is larger than marginal cost for at least one firm.*

Proof. I want to show that: (i) the existence of industry profits larger than non-cooperative level implies that marginal revenues are larger than marginal costs for at least one firm; and

(ii) if at least one firm has marginal revenues larger than marginal costs then industry profits are improved relative to the non-cooperative Nash equilibrium.

Conditions (i) and (ii) can be expressed as:

$$\pi_T(Q_T^*) > \pi_T(Q_T^0) \;\Rightarrow\; \exists i \in 1,\ldots,n: \quad mr_i(q_i^*) > mc \tag{23}$$

$$\exists i \in 1,\ldots,n: \quad mr_i(q_i^*) > mc \Rightarrow \pi_T(Q_T^*) > \pi_T(Q_T^0) \tag{24}$$

respectively, where the superscript "*" represents market outcomes under collusion, and the superscript "^{0}" Cournot equilibria.

(i) Proof by *reductio ad absurdum*. Suppose there is a sustainable equilibrium (identified with the superscript "^{e}") where industry profits are above the non-cooperative Cournot equilibrium and marginal revenues are smaller than or equal to marginal costs for all firms in the industry, i.e.:

$$\pi_T(Q_T^e) > \pi_T(Q_T^0) \quad \wedge \quad mr_i(q_i^e) \le mc, \quad \forall i \in 1,\ldots,n \tag{25}$$

from (25) and (8) it follows that

$$Q_T^e < Q_T^0 \wedge p(Q_T^e) > p(Q_T^0) \tag{26}$$

thus, from Assumption 1(ii) it can be shown that

$$mr_i(q_i^e) = p'(Q_T^e) \cdot q_i^e + p(Q_T^e) > p'(Q_T^0) \cdot q_i^e + p(Q_T^0) \tag{27}$$

It suffices to show that there is one case where (25) is violated. Thus, consider the case of a symmetric equilibrium where

$$q_i^e = \frac{Q_T^e}{n} \wedge q_i^0 = \frac{Q_T^0}{n} \tag{28}$$

it follows that

$$mr_i(q_i^e) > mr_i^0 = mc \tag{29}$$

which contradicts our initial hypothesis.

Q.E.D.

(ii) I show (24) through *reductio ad absurdum*. Suppose that (24) is not true and:

$$\exists i \in 1,...,n: \quad mr_i(q_i^*) > mc \quad \wedge \quad \pi_T(Q_T^*) \le \pi_T(Q_T^0) \tag{30}$$

Note that industry profits in the Cournot Nash equilibrium are given by

$$\pi_T(Q_T^0) = \sum_{i=1}^{n} \pi_i(q_i^o) \tag{31}$$

where total industry output can be rewritten as the sum of the firm own output plus the output of all other firms in the industry

$$Q_T^0 = q_i^0 + Q_{-i}^o \tag{32}$$

and

$$\pi_T^{\prime}(Q_T^0) = p(Q_T^0) + p^{\prime}(Q_T^0)\cdot(q_{-i}^0 + Q_{-i}^0) - mc \tag{33}$$

where $\pi_i(q_i^o)$ is firm *i*'s profit at the non-cooperative equilibrium characterized by (4) for all firms in the industry, and $\pi_T(Q_T^o)$ are the industry non-cooperative Cournot equilibrium profits. From (4) I know that $\pi_i^{\prime}(q_i^0) = 0$. It then follows that:

$$\pi_T^{\prime}(Q_T^0) = \pi_i^{\prime}(q_i^0) + p^{\prime}(Q_T^0)\cdot Q_{-i}^0 = p^{\prime}(Q_T^0)\cdot Q_{-i}^0 < 0 \tag{34}$$

since $p^{\prime}(Q_T^0) < 0$ and $Q_{-i}^0 > 0$. Then, from (8) and (30) it can be shown that

$$\pi(Q_T^*) < \pi(Q_T^0) \Rightarrow Q_T^* > Q_T^0 \Rightarrow p(Q_T^*) < p(Q_T^0) \tag{35}$$

since $Q_T^* > Q_T^0$ then there must exist at least one firm, say firm *i*, that produces at least as much as under Cournot equilibrium, i.e.,

$$\exists i \in 1,...,n: q_i^* \ge q_i^0 \tag{36}$$

Then, from Assumption 1(i), (35), and (36) it follows that the marginal revenue for firm *i*

$$mr(q_i^*) = p(Q_T^*) + p^{\prime}(Q_T^*)\cdot q_i^* < p(Q_T^0) + p^{\prime}(Q_T^*)\cdot q_i^0 \tag{37}$$

since demand is weakly concave (Assumption 1(ii)), it follows that

$$p^{\prime}(Q_T^*) < p^{\prime}(Q_T^0) \tag{38}$$

From (37) and (38) it immediately follows that

$$mr_i(q_i^*) < mr(q_i^0) = mc \tag{39}$$

contradicting our initial hypothesis (30).

Q.E.D.

COROLLARY 2. *The maximization of industry profits (perfect collusion) implies that firm marginal revenues are larger than marginal costs for all firms in the industry, i.e., that each and every firm in the industry colludes.*

Proof. I want to show that: (i) the maximization of industry profits (perfect collusion) implies that firm marginal revenues are larger than marginal costs for all firms in the industry.

Perfect collusion is akin to a situation where the industry oligopoly conduct achieves a performance identical to that obtained if the industry were a monopoly. Condition (i) can be expressed as:

$$\pi_T'(Q_T^*) = 0 \Rightarrow mr_i(q_i^*) > mc \qquad \forall i \in 1,..,n \tag{40}$$

where the superscript "*" represents market outcomes under collusion, and the superscript "^{0}" the Cournot equilibrium. Expression (40) can be rewritten as:

$$p(Q_T^*) + p'(Q_T^*) \cdot Q_T^* = mc \tag{41}$$

or

$$MR_T^* = MC_T = mc \tag{42}$$

where

$$Q_T^* = \sum_{i=1}^{n} q_i^* \tag{43}$$

is the industry optimal output under (perfect) collusion, q_i^* is the cartel output allocation to firm *i*, and MR_T^*, MC_T are the industry marginal revenues and costs, respectively. That is, under perfect collusion a cartel behaves as if it were a monopoly. From (41) and (42) it follows that

$$p(Q_T^*) + p'(Q_T^*) \cdot q_i^* > p(Q_T^*) + p'(Q_T^*) \cdot Q_T^* \quad (44)$$

since demand is downward sloping (Assumption 1(i)) and $q_i^* < Q_T^*$. This expression can be rewritten as

$$mr_i(q_i^*) > MR_T^* \quad (45)$$

where $mr_i(q_i^*)$ designates firm *i*'s marginal revenues with Cournot conjectures at q_i^*.

From (45), (42), and (4) it follows that

$$mr_i(q_i^*) > mc = mr_i^0, \qquad \forall i \in 1,...,n \quad (46)$$

Q.E.D.

Expression (46) indicates that the collusive profit maximization condition negates profit maximization at the firm level, a well known result which Tirole (1988) ascribes to the negative externality between firms: when firms maximize individual profits they take into account the effect of its output change on market price and its own revenues but do not consider the effect on total industry revenues.

Thus, perfect collusion implies that each firm participating in the collusive agreement has (non-cooperative) individual marginal revenues that are larger than the industry marginal revenues under collusion and than the firm marginal costs, i.e., each firm has an incentive to deviate from collusive outcome and to increase output in order to increase profits. In essence, collusion is akin to have each firm in the colluding coalition act as if it had higher marginal costs. Tirole (1988) shows that increases in a firm marginal costs lead the firm to produce less and result in a shift outwards of the residual demand faced by all other firms in the industry. If every firm in the collusion coalition chooses a lower output level in a coordinated manner, the residual demand of each firm shifts outward sufficiently so that as a result each firm has a higher profitability than under the non-cooperative equilibrium.

THEOREM 3. *Collusion occurs if and only if the firm marginal revenue is larger than marginal cost for a sufficiently large subset of the firms in the industry.*

Proof. Follows from Lemma 1, Axiom 1, Theorem 1, and Theorem 2.

4. Collusion test

In Section 3, I showed that if and only if industry profits are larger than Cournot Nash equilibrium profits, then there is collusion (Theorem 1). Alternatively, if and only if the firm colludes, then its marginal revenues with Cournot Nash conjectures are larger than its marginal costs (Theorem 3).

The test can thus be specified as one-tailed hypothesis test either based on Theorem 1:

$$\begin{aligned} &H0:\Pi_{T,t}-\Pi_{T,t}^{0}\leq 0 \\ &HA:\Pi_{T,t}-\Pi_{T,t}^{0}>0 \end{aligned} \quad (47)$$

or alternatively based on Theorem 3:

$$\begin{aligned} &H0:mr_{i,t}-mc_{i,t}\leq 0 \\ &HA:mr_{i,t}-mc_{i,t}>0 \end{aligned} \quad (48)$$

where $\Pi_{T,t},\Pi_{T,t}^{0}$ are actual total industry profits and hypothetical total industry Cournot Nash equilibrium profits in period *t*, respectively, and $mr_{i,t}, mc_{i,t}$ are firm *i*'s marginal revenues (with Nash conjectures) and marginal costs in period *t*, respectively.

In both specifications, rejection of the null hypothesis indicates that the hypothesis of non-collusive behavior should be rejected. The first specification allows the rejection of non-collusive behavior at the industry level, for example, using industry level time series data. The second specification allows the rejection of non-collusive behavior at the firm level, for example, using firm level time series data. Note that the purpose of the test is to quickly flag instances of suspected collusion for further investigation by authorities, which would then determine whether it is tacit or explicit collusion. The test does not, by itself, constitute evidence of explicit collusion.

5. Open issues

The discussion in this section identifies open issues for further research, and discusses possible approaches to implementing the proposed collusion tests.

5.1. Tacit versus explicit collusion

Consider the collusion test specification given by (48). If the null hypothesis of no collusion is rejected with a greater degree of statistical significance, it is likely that for the observations of the sample, firm marginal revenue is much larger than its marginal cost, i.e., the observations underlying the empirical results likely fall on right tail of the distribution. For those observations, if the firm marginal revenue is much larger than its marginal cost, then the incentive to secretly violate the collusive agreement is large. Tacit collusion is more difficult to maintain if the incentive to violate the agreement is large. Thus, when the null hypothesis is rejected at the one percent level, we are likely to find a much smaller proportion of tacit collusion cases, than when the null hypothesis is rejected at the five or ten percent level.

Likewise, if marginal revenue is only slightly larger than marginal cost, then the gains from collusion are likely to be relatively small. On the one hand, for firms engaging in explicit collusion, profits from collusion must be larger than the expected costs of explicit collusion (see proof Lemma 1). On the other hand, better coordination is likely to allow explicit colluding coalitions to increase market power by more than feasible for tacit colluding coalitions, and thus in larger differences between each colluding firm marginal revenues and marginal costs. Thus, the proportion of explicit collusion cases, when marginal revenue is only slightly above marginal cost, is likely to be much smaller than that when marginal revenue is much larger than marginal costs. If indeed so, the proposed test would have attractive properties, as type I errors would likely be tacit collusion cases, which are not relevant for regulatory purposes.

5.2. Data issues

The main issue is what variables can be more easily obtained by regulatory authorities in order to detect instances of collusion. The data required for the tests suggested by either Theorem 1 or 3 are difficult to obtain, more so than data on prices and marginal costs as used, for example, in the Hall (1988) analysis of price-cost margins for different US industries. For both tests, there is the issue of the definition of the relevant market, and data availability in the necessary detail, particularly for multi-product firms.

A test based on comparison of industry profits with Cournot Nash equilibrium profits (Theorem 1) would face a number of challenges. First, there is a difference between economic and accounting profits. Second, there would be fewer observations and probably more missing observations, as the test would be based on a comparison of industry level data. Still, even if, for example, it is not possible to obtain data for small firms, Corollary 1 indicates that the same test could be applied for any subset of the industry. Finally, the reference industry profits under Cournot Nash equilibrium, which would be used as basis of comparison, cannot be observed. A possible approach would be to use the fact that firm market shares, in a Cournot Nash equilibrium, provide information about each firm's marginal costs.

A test based on the comparison of marginal revenues and marginal costs also faces significant data challenges. Both variables are not directly observable. Nonetheless, prior literature has looked at price cost margins (Cowling and Waterson, 1976; Hall, 1988; Parker and Röller, 1997), and therefore different approaches have been proposed to measure marginal costs. Structural conduct performance paradigm empirical studies used accounting measures of marginal costs. One possible approach in this line, assuming the availability of product line accounting revenues, costs, and output levels, could be to derive average incremental marginal revenues and marginal costs from quarterly or yearly sales and cost data. An alternative approach, would be to follow methodologies of the New Empirical Industrial Organization literature (Hall, 1988; Bresnahan, 1989; Parker and Röller, 1997) that attempt to estimate parameters of the firm's supply response (Corts, 1999).

6. Conclusion

It is well known that collusion outcomes are unnatural in that colluding firms act contrary to the economic incentives they face individually. Colluding firms have the incentive to secretly violate the collusive agreement and one of the main issues associated with the sustainability of collusion is to prevent such violations of the collusive agreement from occurring (Stigler, 1964).

The proposed test of collusive conduct, based on the analysis of firm incentives, seeks to detect firm cooperative behavior not consistent with the perfect Nash equilibrium profit maximizing condition, a situation that occurs under both tacit and explicit collusion. It has several advantages. For one, measures of market performance, such firm and industry price-cost margins are not directly relevant to the test, thus it does not penalize efficient firms (Demsetz, 1973). Market structure and the degree of contestability of the market similarly do not affect the efficacy of the proposed test (Cabral, 2000; Baumol *et al*, 1982), since the test analyzes firm incentives for a given firm market structure, and firm conduct consistent with the contestable market hypothesis will result in the non-rejection of the null hypothesis of no collusive behavior. Further, in the context of dynamic competition, the test may detect instances of tacit collusion where firms do not optimize profits for fear of retaliatory action by competitors. Finally, the test does not depend on the estimation of a conduct parameter that in effect measures how the industry conduct is correlated to that of industries under competitive or monopoly equilibria (and to price-cost margins). The test simply measures whether the firm has an incentive to deviate from its current output decision.

The paper raises interesting questions that will hopefully be subject of further research. These include, among others, the analysis of the Bertrand dual case and the consideration of product differentiation.

7. References

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