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Complementarities and costly investment in a growth model

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Abstract We develop a one-sector growth model whose key aspect is the combination of the assumptions of internal costly investment in capital and complementarities between capital goods in the production function. Despite the nonlinearity generated by these assumptions, the model delivers a single equilibrium.

Keywords Costly investment · Complementarities · Economic growth

JEL Classification O30 · O40 · O41

1 Introduction

We introduce internal costs of investment into the R&D-based growth literature, and develop a one-sector growth model which combines the assumption of internal adjustment costs with the assumption of complementarities between capital goods in the production function.

In traditional growth theory, capital accumulation is determined residually as the difference between production and consumption, and is costless. But, as argued by Benavie et al. (1996) and Romer (1996), growth models should treat investment as a decision variable of the firm. This requires costs to accumulating capital.

As reviewed by Turnovsky (1995), the implications of the introduction of an adjustment costs function in macroeconomics, and in particular its link with the Tobin-*q*, were first developed by Summers (1981) and Hayashi (1982), and further explored by Abel (1982) and Abel and Blanchard (1983) in more specific models. Convex adjustment costs have played a crucial role in the development of open economy

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models with intertemporal optimization. Cohen (1993) and Van Der Ploeg (1996) provide examples of such frameworks. A further example, in a different context, is Benavie et al. (1996) who introduce investment costs into a stochastic growth model in order to analyse the effects of fiscal policy on the rate of economic growth.

In order to model the existence of internal investment costs, we use an application of Hayashi's (1982) cost of investment framework to a continuous time context, as done by Benavie et al. (1996), Cohen (1993), Van Der Ploeg (1996). While these models apply AK production functions, our proposed contribution is to introduce internal investment costs into a one-sector R&D-based growth model.

The developed one-sector framework allows for the inclusion of R&D investment in total capital investment. Hence it allows us to capture internal R&D investment costs within the total internal investment costs.

The proposed model also assumes that capital goods are complementaries in the production function. As Matsuyama (1995) writes, the notion that complementarities are an essential feature in explaining economic growth, business cycles and underdevelopment has long been conveyed by authors such as Hicks (1950), Kaldor (1985), Myrdal (1957). Bryant (1983) also stresses the importance of complementarities between capital goods in production. The presence of complementarities means that if the number of complementary goods increases, the production of a capital good will increase. In turn, by increasing its output, a producer of a capital good raises the demand for its complementary goods. Personal computers, printers and communication networks constitute familiar examples of capital goods that are complements.

In order to capture the presence of complementarities between capital goods in the production function, our model builds on Evans et al. (1998). Evans et al. (1998) set up a two-sector framework with external costs of investment, as consumption and total capital are produced under different technologies. Further, Evans et al. (1998) specify the price of capital in terms of consumption as an analytically-non-observable function. With the purpose of introducing internal investment costs into the R&D-based growth literature, we depart from Evan et al.'s (1998) two-sector structure and also eliminate the analytically-non-observable external investment cost. Following Rivera-Batiz and Romer (1991), we specify a one-sector framework in which consumption and total capital are produced with the same technology. We assume internal costs of investment by adopting an analytically-observable investment cost function, due to Hayashi (1982).

Despite the nonlinearity generated by the combination of costly investment and complementarities, the model delivers a unique equilibrium.

The paper is organised as follows. Section 2 provides a motivation for the adopted investment cost specification. Section 3 presents the proposed general equilibrium growth model and its main results. Section 4 concludes.

2 Investment cost specification

When firms face zero capital accumulation costs, for a given level of output and a linearly homogeneous production function, the optimal level of capital stock can be determined, but the rate of optimal investment is indeterminate. This indeterminacy of investment led to the modified investment theory, where the firm maximises the present discounted value of its cash flows subject to capital installation costs.

Assuming Hayashi's (1982) investment cost specification, and zero capital depreciation, the installation of $I(t) = \overset{\bullet}{K}(t)$ new units of capital requires the firm to spend an amount given by $J(t) = I(t) + \frac{1}{2}\theta\frac{I(t)^2}{K(t)}$, where the installation cost function is $C(I(t), K(t)) = \frac{1}{2}\theta\frac{I(t)^2}{K(t)}$. The *current-value Hamiltonian* for the firm's maximisation problem is:

$$H(t) = F(K(t), \overline{L}) - I(t) - \frac{1}{2}\theta\frac{I(t)^2}{K(t)} + q(t)(I(t) - \overset{\bullet}{K}(t)).$$

The purchase price of capital is assumed to be $P_K = 1$, hence the ratio of the market value of a unit of capital, q, to its replacement cost is equal to q. This ratio is known as Tobin's (1969) *marginal q*. In turn, the ratio of the market value of the firm to the replacement cost of its total capital stock is *average q*.

It is *marginal q* that is relevant to investment. However, since only *average q* is observable, empirical studies have used *average q* as an approximation to *marginal q*. With Hayashi's (1982) investment cost function, this empirical issue is resolved because *marginal q* and *average q* are equal.

3 Specification and results of the model

3.1 Consumption side

Infinitely lived homogeneous consumers maximise, subject to a budget constraint, the discounted value of their representative utility:

$$\max_{C(t)} \int_0^\infty e^{-\rho t}\frac{C(t)^{1-\sigma}}{1-\sigma}dt$$

$$s.t. \quad \overset{\bullet}{B}(t) = rB(t) + w(t) - C(t),$$

where variable $C(t)$ is consumption in period t, ρ is the rate of time preference and $\frac{1}{\sigma}$ is the elasticity of substitution between consumption at two periods in time. Variable $B(t)$ stands for total assets, r is the interest rate, $w(t)$ is the wage rate, and it is assumed that households provide one unit of labour per unit of time.

The transversality condition of this optimisation problem is $\lim_{t\to\infty}\mu(t)B(t) = 0$, where $\mu(t)$ is the shadow price of assets, and consumption decisions are given by the familiar Euler equation:

$$g_c = \frac{\overset{\bullet}{C}}{C} = \frac{1}{\sigma}(r - \rho). \tag{1}$$

3.2 Production side

There are three productive activities: final good production, capital goods production and invention of new capital goods, that is, research and development (R&D) activities. With the purpose of introducing internal costly investment, we assume that final-good producers incur a Hayashi's (1982) investment cost when accumulating total capital which is composed by physical capital and inventions.

The presence of complementarities is modelled by building on Evans et al. (1998), who assume complementarities between capital goods in the production function.

3.2.1 Complementarities between capital goods

Building on Evans et al. (1998), we assume that the final good Y is produced using as inputs labour L, assumed constant, and a number A of differentiated durable capital goods i, each produced in quantity x_i. Capital goods enter complementarily in the production function:

$$Y(t) = L(t)^{1-\alpha} \left(\int_0^{A(t)} x_i(t)^{\gamma} di \right)^{\phi}, \quad \phi > 1, \quad \gamma\phi = \alpha,$$

where the assumption $\phi > 1$ is made so that capital goods are complementary to one another, that is, an increase in the quantity of one good increases the marginal productivity of the other capital goods. The restriction $\gamma\phi = \alpha$ is imposed in order to preserve homogeneity of degree one. To solve the model for a constant growth rate, we impose:

$$\xi = \frac{\phi - 1}{1 - \alpha}.$$

The second productive activity concerns the production of physical machines for each of the already invented types of capital goods. Assuming that it takes one unit of physical capital to produce one physical unit of any type of capital good, physical capital K is related to the capital goods by the rule:

$$K(t) = \int_0^{A(t)} x_i(t) di.$$

Turning to R&D activities, following Rivera-Batiz and Romer (1991), we assume that new designs are invented with the same technology as that of the production of the final good and of capital goods. We further assume that the invention of patent i requires $P_A\, i^{\xi}$ units of foregone output, where P_A is the fixed price of one new design in units of foregone output, and i^{ξ} represents an additional cost of patent i in terms of foregone output, meaning that there is a higher cost for designing goods with a higher index. This extra cost is introduced in order to avoid explosive growth.

Total investment in each period is then given by:

$$\dot{W}(t) = \dot{K}(t) + P_A \dot{A}(t) A(t)^{\xi},$$

where $\dot{K}(t)$ represents investment in physical capital, and $P_A \dot{A}(t) A(t)^{\xi}$ represents investment in the invention of new designs.

Total capital $W(t)$ is equal to:

$$W(t) = K(t) + P_A \frac{A(t)^{\xi+1}}{\xi + 1},$$

and it accumulates according to:

$$\dot{W}(t) = Y(t) - C(t). \tag{2}$$

Final good producers are price takers in the market for capital goods. In equilibrium they equate the rental rate on each capital good with its marginal productivity, so the demand curve faced by each capital good producer is:

$$R_j(t) = \phi \gamma L^{1-\alpha} x_j(t)^{\gamma - 1} \left(\int_0^{A(t)} x_i(t)^{\gamma} di \right)^{\phi - 1}.$$

The symmetry of the model implies that $R_j(t) = R(t)$, and $x_j(t) = x(t)$. Hence, the production function for aggregate output can be rewritten as:

$$Y = L A^{1+\xi} \left(\frac{\alpha}{R} \right)^{\frac{\alpha}{1-\alpha}} = BW, \tag{3}$$

where B, the marginal productivity of total capital, is constant.

3.2.2 Internal costly investment

We assume that final good producers own total capital and incur an internal investment cost. We assume that, with zero capital depreciation, installing $I(t) = \dot{W}(t)$ new units of total capital requires the final good firms to spend an amount given by:

$$J(t) = I(t) + \frac{1}{2} \theta \frac{I(t)^2}{W(t)},$$

where $C(I(t), W(t)) = \frac{1}{2} \theta \frac{I(t)^2}{W(t)}$ represents the Hayashi's (1982) installation cost.

Final good firms choose the investment rate so as to maximise the present discounted value of their cash flows. Having in mind Eq. (3), the *current-value Hamiltonian* is:

$$H(t) = BW(t) - I(t) - \frac{1}{2}\theta\frac{I(t)^2}{W(t)} + q(t)(I(t) - \overset{\bullet}{W}(t)),$$

where $q(t)$ is the market value of capital and the transversality condition of this optimization problem is $\lim_{t\to\infty} e^{-rt}q(t)W(t) = 0$.

Recalling that the growth rate of output is $g = \frac{I}{W}$, the first-order condition is equivalent to:

$$q = 1 + \theta g, \tag{4}$$

and the co-state equation is equivalent to:

$$\frac{\overset{\bullet}{q}}{q} = r - \frac{B + \frac{1}{2}\theta g^2}{q}. \tag{5}$$

The problem is solved for its balanced growth path solution, which implies that q must be constant. Hence, Eq. (5) becomes:

$$q = \frac{B + \frac{1}{2}\theta g^2}{r}.$$

We turn now to the capital good firms production decisions. Once invented, the physical production of each unit of the specialised capital good requires one unit of capital. So, in each period the monopolistic capital good producer maximises its profits, taking as given the demand curve for its good:

$$\max_{x_j(t)} \pi_j(t) = R_j(t)x_j(t) - rqx_j(t),$$

which leads to the markup rule:

$$R_j = \frac{rq}{\gamma}.$$

At time t, in order to enter the market and produce the Ath capital good, a firm must spend up-front an amount given by $P_A A(t)^{\xi}$, where, as mentioned earlier, P_A is the fixed price of one new design in units of foregone output, and i^{ξ} represents an additional cost of patent i in terms of foregone output. Hence, the dynamic zero-profit condition is:

$$P_A A(t)^{\xi} = \int_t^{\infty} e^{-r(\tau-t)}\pi_j(\tau)d\tau,$$

which, assuming no bubbles, is equivalent to:

$$\xi g_A = r - \frac{\pi}{P_A A^{\xi}}. \tag{6}$$

In a balanced growth path, x grows at the rate $\frac{\dot{x}}{x} = \xi g_A$ and output grows at the rate $g_y = (1+\xi)g_A$. Rearranging the expression for profits as $\pi = \left(\frac{1-\gamma}{\gamma}\right) rqLA^{\xi} \left(\frac{\alpha\gamma}{rq}\right)^{\frac{1}{1-\alpha}}$, equation (6) becomes:

$$g = \frac{1+\xi}{\xi}\left[r - \frac{\Omega}{(rq)^{\frac{\alpha}{1-\alpha}}}\right], \quad \Omega = \frac{\left(\frac{1-\gamma}{\gamma}\right) L\,(\alpha\gamma)^{\frac{1}{1-\alpha}}}{P_A}. \tag{7}$$

Equation (7) unites the equilibrium balanced growth path pairs (r, g) on the production side of this economy. We call it Technology curve, after Rivera-Batiz and Romer (1991).

3.3 General equilibrium

The capital accumulation Eq. (2) tells us that a constant growth rate of total capital implies that consumption grows at the same rate as output. This means that, as labour is constant, the per-capita economic growth rate is given by:

$$g_c = g_y = g_k = g_w = g = (1+\xi)g_A.$$

The general equilibrium solution is obtained by solving the system of the two Eqs. (1) and (7), in two unknowns, r and g. Recalling Eq. (4), the system to be solved is:

$$\begin{cases} g = \frac{1}{\sigma}(r-\rho) \\ g = \frac{1+\xi}{\xi}\left[r - \frac{\Omega}{(r+r\theta g)^{\frac{\alpha}{1-\alpha}}}\right] \end{cases}, \quad r > g > 0,$$

where $\Omega = \frac{\left(\frac{1-\gamma}{\gamma}\right)L(\alpha\gamma)^{\frac{1}{1-\alpha}}}{P_A}$.

The restriction $r > g > 0$ is imposed so that present values will be finite, and also so that our solution(s) have positive values for the interest rate and the growth rate.

While the Euler equation is linear and positively sloped in the space (r, g), the Technology curve is nonlinear, as shown in the Appendix. Since the nonlinearity of the Technology curve does not allow for the analytical derivation of the equilibrium solution(s), we resort to solving the system through a numerical example. The chosen values for our parameters are:

$$\begin{aligned} &\sigma = 2; \quad \rho = 0.02; \quad \alpha = 0.4; \quad \gamma = 0.1; \\ &\xi = 5; \quad L = 1; \quad \theta = 3; \quad P_A = 5, \end{aligned}$$

where the values for α, γ and consequently $\phi = \frac{\alpha}{\gamma}$ and $\xi = \frac{\phi-1}{1-\alpha}$ are the same as those used by Evans et al. (1998) in their numerical example. The values for the preference parameters σ and ρ are in agreement with those found in empirical studies such as

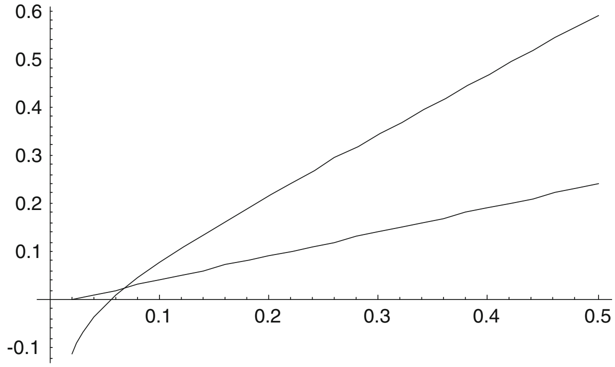

Fig. 1 General equilibrium solution

Barro and Sala-i-Martin (1995). Population is often chosen to have unity value. And the values for θ and P_A were chosen so as to give us realistic values for the equilibrium growth rate and interest rate.

Although the Technology curve is nonlinear, a unique solution is found. For the adopted parameter values, it is:

$$g = 0.024; \quad r = 0.068.$$

Figure 1, with r on the horizontal axis and g on the vertical axis, helps us visualise this economy's balanced growth path general equilibrium solution.

Proposition 1 *A unique solution to this growth model with complementarities and costly investment exists for* $\sigma > 1$ *and* $\Omega^{1-\alpha} > \rho$.

Proof Defining two new variables and rewriting our system, we can show that the proposed model does have a unique solution. Our new variables are:

$$Y = \theta g; \quad Z = r(1 + \theta g),$$

which allows us to rewrite the system as:

$$\begin{cases} Z^{\beta} = \frac{\lambda}{Y+\mu} \\ Z = \frac{\sigma}{\theta}\left(Y+1\right)\left(Y+\eta\right) \end{cases},$$

where $\beta = \frac{\alpha}{1-\alpha}$, $\lambda = \frac{\theta\Omega}{\sigma - \frac{\xi}{1+\xi}}$, $\mu = \frac{\rho\theta}{\sigma - \frac{\xi}{1+\xi}}$, $\eta = \frac{\rho\theta}{\sigma}$. Our restrictions become:

$$Y > 0; \quad Z > \frac{1}{\theta} Y(Y+1).$$

To ensure that $r > g$, we impose $\sigma > 1$ so that the Euler equation (1) lies above the 45° line. This implies that λ, μ and η are all positive. Hence the first equation of the rewritten system defines a strictly decreasing curve $Y \mapsto Z(Y)$ from $Z(0) = \left(\frac{\Omega}{\rho}\right)^{\frac{1}{\beta}}$ to $Z(\infty) = 0$, while the second equation defines a strictly increasing curve $Y \mapsto Z(Y)$ from $Z(0) = \rho$ to $Z(\infty) = \infty$. Hence the system has a unique solution in the region $Y > 0$ iff $\Omega > \rho^{\beta+1}$(which is equivalent to $\Omega^{1-\alpha} > \rho$). The second restriction is also met because $Z = \frac{\sigma}{\theta}\,(Y+1)\,(Y+\eta) > \frac{1}{\theta}Y(Y+1)$. □

It is interesting to note that, as Matsuyama (1995, 1997) analyses, the presence of complementarities in a growth model is generally associated with multiple equilibria, as it makes the model nonlinear. That is for instance the case with Evans et al. (1998) model. For Evans et al. (1998), multiple equilibria is the intended result, as the authors wish to explain growth cycles. Hence they combine complementarities with external costs of investment in a two-sector framework, and specify a nonlinear trade-off between consumption and capital accumulation, so as to obtain a nonlinear Technology curve.

The model presented here was developed with the purpose of analysing the impact on growth of internal costs of investment in an R&D-based environment with complementarities, in which R&D investment is part of total capital investment. We find that the combination of the assumption of complementarities between capital goods in the production function and the assumption of internal costly investment in capital also gives rise to a nonlinear Technology curve. However this model delivers a unique equilibrium.

4 Concluding remarks

We have developed a one-sector growth model which combines the assumption of internal investment costs with the assumption of complementarities between capital goods in the production function.

We introduce two novelties. First, internal costly investment is new to R&D-based growth literature. Second, despite the nonlinearity generated by the presence of complementarities combined with the existence of internal costs of investment, our model delivers a single equilibrium.

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Appendix

In order to analyse the shape of the Technology curve (7), and as it is impossible to isolate r on one side of the equation, we rewrite it as $F(r, g) = 0$ and apply the implicit function theorem, so as to obtain, in the neighbourhood of an interior point of the function, the derivative $\frac{dr}{dg}$:

$$F(r, g) = \xi g - (1+\xi)\,r + (1+\xi)\,\Omega r^{\frac{-\alpha}{1-\alpha}}\,(1+\theta g)^{\frac{-\alpha}{1-\alpha}} = 0,$$

which leads to:

$$\frac{dr}{dg}=-\frac{\frac{dF(r,g)}{dg}}{\frac{dF(r,g)}{dr}}=\frac{\xi-\left(\frac{\alpha}{1-\alpha}\right)\theta\left(1+\xi\right)\Omega r^{\frac{-\alpha}{1-\alpha}}\left(1+\theta g\right)^{\frac{-1}{1-\alpha}}}{\left(1+\xi\right)+\left(\frac{\alpha}{1-\alpha}\right)\left(1+\xi\right)\Omega r^{\frac{-1}{1-\alpha}}\left(1+\theta g\right)^{\frac{-\alpha}{1-\alpha}}}.$$

Hence, our nonlinear Technology curve is positively sloped when:

$$r^{\frac{-\alpha}{1-\alpha}}\left(1+\theta g\right)^{\frac{-1}{1-\alpha}}<\frac{\xi}{\left(\frac{\alpha}{1-\alpha}\right)\theta\left(1+\xi\right)\Omega},$$

and negatively sloped otherwise.

Replacing the expression for g given by the Euler equation in the Technology curve, we obtain the equilibrium expression for r:

$$r\left(\frac{1}{\sigma}-\frac{1+\xi}{\xi}\right)+\frac{\left(\frac{1+\xi}{\xi}\right)\Omega}{\left[\frac{\theta}{\sigma}r^{2}+\left(1-\frac{\theta\rho}{\sigma}\right)r\right]^{\frac{\alpha}{1-\alpha}}}=\frac{\rho}{\sigma}.$$

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