International Journal of Industrial Organization 28 (2010) 669–681

Contents lists available at ScienceDirect

International Journal of Industrial Organization

journal homepage: www.elsevier.com/locate/ijio

Pricing with customer recognition☆

Rosa-Branca Esteves *

Department of Economics and NIPE, University of Minho, Campus de Gualtar, 4710 Braga, Portugal

ARTICLE INFO

Article history:
Received 17 May 2008
Received in revised form 8 March 2010
Accepted 9 March 2010
Available online 19 March 2010

JEL classification:
D43
L13

Keywords:
Competitive behaviour-based price discrimination
Discrete distribution of consumer preferences
Economic effects

ABSTRACT

This article studies the dynamic effects of behaviour-based price discrimination and customer recognition in a duopolistic market where the distribution of consumers' preferences is discrete. Consumers are myopic and firms are forward looking. In the static and first-period equilibrium firms choose prices with mixed strategies. When price discrimination is allowed, forward-looking firms have an incentive to avoid customer recognition, thus the probability that both will have positive first-period sales decreases as they become more patient. Furthermore, an asymmetric equilibrium sometimes exists, yielding a 100–0 division of the first-period sales. As a whole, price discrimination is bad for profits but good for consumer surplus and welfare.

1. Introduction

The increasing use of the Internet and the development of more sophisticated methods for acquiring, storing and analysing consumer information have dramatically improved the capability of sellers to learn the consumers types or preferences through the observation of their past behaviour, and to set prices accordingly in subsequent periods. As customer recognition and behaviour-based price discrimination (BBPD) are becoming increasingly prevalent, a good economic understanding of the profit, consumer surplus and welfare implications of this price discrimination practice needs to be founded on a good economic understanding of the market in which it is implemented.

The aim of this paper is to investigate the competitive and welfare effects of BBPD in markets where the distribution of consumer types is discrete and each firm follows in equilibrium a mixed pricing strategy as an attempt to prevent the rivals from systematically predicting its price.[1] This pricing strategy seems to be in accordance with several studies showing that random pricing is a feature of online markets[2] and may be the result of retailers heterogeneity with respect to brand loyalty, trust, and awareness (Brynjolfsson and Smith (2000a)). The stylised model addressed in this paper brings new insights to the literature in the field and helps establish the idea that some of the competitive effects of BBPD and customer recognition do depend on what is learned about consumer demand, which in turn depends on the nature of preferences.

The paper considers a repeated interaction model with myopic consumers and forward-looking firms. Firms A and B market their goods directly to consumers who are either loyal (to a specific degree) to one firm or the other. To motivate the distribution of consumer preferences consider the following example. Suppose there are two online firms: Amazon (A) and Barnes&Noble (B). Both firms know that half of consumers have a relative preference for A while the remaining have a relative preference for B. The disutility of not buying

☆ This paper is based on my PhD thesis (Oxford University). I am extremely grateful to Mark Armstrong (my supervisor), the Editor Bernard Caillaud and an anonymous referee for comments that have significantly improved the paper. Thanks are also due to Paul Klemperer, Robin Mason, Miguel Villas-Boas, Maria-Angeles de Frutos and Francisco Cruz. Financial support from Fundação para a Ciência e a Tecnologia is gratefully acknowledged. Any errors are my own.

* Tel.: +351 253601932.
E-mail address: rbranca@eeg.uminho.pt.

[1] In their survey on BBPD and customer recognition, in Section 3.1. (on BBPD under competition with two-periods and short-term contracts), Fudenberg and Villas-Boas (2006) discuss three papers: the Fudenberg and Tirole (2000) and its extension by Chen and Zhang (2009) (previous version 2004) and chapter 3 of Esteves (2004) (a previous version of the current paper) to other distributions of consumer types, where other insights emerge. In page 35 they point out that concavity does not seem to be implied by the MHR condition and when it fails it may be interesting to work with discrete types as in Esteves and Chen and Zhang.

[2] See for instance Brynjolfsson and Smith (2000a) and Baye et al. (2006a).

0167-7187/$ – see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.ijindorg.2010.03.008

the most preferred brand is exogenously given by $\gamma>0$.[3] So, a consumer that prefers Amazon buys from Amazon if its price is not undercut by more than γ by Barnes&Noble. This means that even though firms may have some advantage over their competitors due to brand loyalty, all consumers may, nevertheless, be induced to switch.[4] There are two periods so, being permitted, price discrimination can only occur in the second period when firms have learnt the consumer types by observing their first period choices.

In order to measure the dynamic effects of BBPD two static benchmarks are analysed (Section 3). In the first, price discrimination is not permitted, either because consumers are anonymous or because price discrimination is illegal. In the second, consumers are non-anonymous and price discrimination is permitted. The results derived in both benchmarks show that firms are clearly worse off when they have the required information for price discrimination.

The two-period interaction game with price discrimination based on customer recognition is presented in Section 4. In contrast to the extant models with a continuous distribution of consumer types (e.g. Chen (1997), Fudenberg and Tirole (2000) and Villas-Boas (1999)), where the equilibrium is in pure strategies, the model yields a first-period equilibrium where each firm follows a mixed pricing strategy.[5]

An important theme of the paper is to highlight the fact that firms may eschew learning the consumer types as a way to avoid subsequent price discrimination and a less favourable competitive outcome. When initial market shares are asymmetric (100–0 division) nothing is learnt about consumers, and subsequent prices and profits are higher. In contrast, with symmetric market shares, consumer types are *fully* revealed and second-period prices and profits fall. Consequently, it is shown that forward-looking firms have an incentive to strategically reduce the probability of sharing the market in period 1 (Corollary 3). This strategic effect gives rise to new issues regarding the effect of BBPD on first-period prices. In contrast to Fudenberg and Tirole (2000) firms price *below* the static or no-discrimination levels in period 1 (Proposition 5).

Given that the model predicts the existence of a bias towards asymmetric outcomes in period 1, Section 6 investigates the circumstances in which an *asymmetric* pure strategy equilibrium could exist in the initial period, being the market served by the same firm. Here it is found that there is sometimes an asymmetric equilibrium in the first period, where one firm sets a low price and captures all consumers while the rival finds not profitable to match the low price firm because its profits will then be low in the second period (Proposition 7). This finding suggests that in a many-period game the uniform pricing could be sustained without any explicit collective action.

Finally, the paper investigates the welfare results of BBPD in markets where firms set random prices (Section 7). A common prediction in the existing literature on BBPD is that price discrimination can be welfare reducing due to excessive switching (e.g. Chen (1997), Fudenberg and Tirole (2000)). Another important theme of the paper is to show that as random pricing tends to generate some inefficient shopping, price discrimination can increase efficiency and enhance consumer surplus and social welfare.

[3] In a location model γ can be used as a transportation cost. As in Shilony (1977) consumers can purchase costlessly from the neigbourhood firm but incur a transport cost to go to the more distant firm.

[4] In a recent study Brynjolfsson and Smith (2000b) have found that Amazon customers are willing to pay up to 5–8% more before they consider switching to another seller.

[5] Mixed pricing is also obtained in Chen and Zhang (2009) and Esteves (2009a). However, in these price discrimination models each firm has a captive group of consumers and firms only compete for the price-sensitive consumers.

1.1. Related literature

This paper is mainly related to two strands of the literature. One is the literature on competitive price discrimination,[6] especially the literature on behaviour-based price discrimination and customer recognition.[7] The other is the literature on mixed pricing in oligopoly.[8]

The model addressed in this paper is closely related to Shilony's (1977) one-period oligopolistic model where the distribution of consumer types is discrete. In the location interpretation of his model, consumers can purchase costlessly from neighbourhood firms, but incur a transport cost $\gamma>0$ when buying from more distant firms.[9] In this setting, he shows that the unique equilibrium is in mixed strategies. The same happens in models where some consumers are captive or uninformed while others are shoppers or fully informed (e.g. Varian (1980), Narasimhan (1988)). However, while in Varian (1980) and Narasimhan (1988) each firm has a captive group of customers and firms only compete for price-sensitive customers, in Shilony's model no firm has a group of captive customers and all consumers may be induced to switch. As a result, while in Varian and Narasimhan firms may sometimes set in equilibrium the monopoly price, the same is not true in Shilony. In this way, an interesting finding in Shilony is that firms can attain higher profits in the mixed strategy equilibrium.

Recent work on Internet pricing has shown that random pricing is a common practice for sellers operating in these markets (e.g. Baye et al. (2004, 2006a)). As pointed out by Baye, et al. (2007, p.13), "one way to keep rivals from responding is to introduce an element of randomness into your pricing strategy. By being unpredictable, your rivals cannot systematically undercut your price. "They also highlight that "such a strategy, which reduces the ability of competitors to both anticipate and respond to a price cut, can generate top line growth and raise profits as well." (p.16)

Regarding the literature on competitive price discrimination, this paper is related to those models where, in the terminology of Corts (1998), the market exhibits *best-response asymmetry*.[10] In these models profit will typically decrease when price discrimination is practiced. A useful model for understanding the profit effects of price discrimination in markets with best-response asymmetry is by Thisse and Vives (1988). There are two firms located at the extremes of the segment [0, 1]. Consumers are uniformly distributed in the line segment and firms can observe the location (or brand preference) of each individual consumer and price accordingly. The strong (close) market for one firm is the weak (distant) market for the other firm. In this setting they show that price discrimination intensifies competition, *all* prices fall as well as profits. The finding that firms might be worse off when they engage in price discrimination is one of the key differences between monopoly and competitive price discrimination. If we ignore commitment issues, a monopolist is better off when it uses price discrimination. Although with competition price discrimination is a dominant strategy for each firm, for given prices offered by its rival, when all firms follow the same strategy they might find themselves in the classic prisoner's dilemma.[11]

[6] Comprehensive surveys on competitive price discrimination are presented by Armstrong (2006) and Stole (2007).

[7] For a comprehensive review on BBPD see Fudenberg and Villas-Boas (2006) and Esteves (2009c).

[8] See Baye, et al. (2006b) for a comprehensive survey on Information, Search and Price Dispersion.

[9] A similar modelling approach is followed by Padilla (1995) and with asymmetric loyalties by Raju, et al. (1990).

[10] The market exhibits best-response asymmetry when one firm's "strong" market is the other's "weak" market. In the literature of price discrimination, a market is designated as "strong" if in comparison to uniform pricing a firm wishes to increase its price there. The market is said to be "weak" if the reverse happens.

[11] Esteves (2009b) extends the Thisse and Vives model to a two-dimensional differentiation model and shows that price discrimination might not necessarily lead to the prisoner's dilemma result. This happens when firms observe the location of consumers in the less differentiation dimension and price discriminate accordingly while they remain ignorant about their location in the more differentiated dimension.

The paper has important connections with the literature on BBPD in which firms and consumers interact more than once and firms may be able to learn the consumers' types by observing their past choices and price differently towards them in subsequent periods. In the switching costs approach, consumers initially view the two firms as perfect substitutes; but in the second period they face a switching cost if they change supplier. In this setting, purchase history discloses information about exogenous switching costs (e.g. Chen (1997) and Taylor (2003)). In the brand preferences approach (e.g. Villas-Boas (1999), Fudenberg and Tirole (2000)), purchase history discloses information about a consumer's exogenous brand preference for a firm. Although the framework of competition differs in the two approaches their predictions have some common features. First, as in the present model, when price discrimination is permitted, firms offer better deals to the competitor's consumers than to its previous customers. Second, because both firms have symmetric information for price discrimination purposes and there is best-response asymmetry, firms are *worse off* if they use price discrimination.[12,13] Third, there is socially excessive switching between firms. Nonetheless, important differences arise in both approaches when taking into account the effects of poaching on initial prices. While in the brand preferences approach when BBPD is permitted initial prices are high and then decrease, in the switching costs approach the reverse happens.

Another closely related papers are Chen and Zhang (2009) and Esteves (2009a). The first, investigates the profitability issue of BBPD in the brand preference approach using a discrete version of the Fudenberg and Tirole (2000) model. As in Narasimhan (1988), each firm has an exogenous captive segment of the market, and they compete for the remaining consumers, who are price-sensitive. The equilibrium is in mixed strategies and they show that when only one firm can engage in BBPD both firms can attain higher profits with discrimination. Esteves (2009a) investigates the effects of BBPD in a market where firms need to invest in informative advertising to generate demand. Consumers are initially uninformed about the firms' existence and price. However, after advertising decisions have been made, some consumers are endogenously locked-in with a certain firm while others are shoppers because they become aware of both firms' existence and price. The distribution of consumer types is discrete and the equilibrium is also in mixed strategies. The paper shows that only the high price firm in period 1 will have the required information to engage in price discrimination. This acts to soften price competition in both periods and profits rise if price discrimination is used.

In contrast to Chen and Zhang (2009) and Esteves (2009a) where some consumers are captive and others are shoppers, here although consumers have a preference for one of the firms all of them may be induced to switch. If price discrimination is permitted it will only occur when both firms share the market in period 1. In this case, *both* firms will practice price discrimination in period 2 and they will be worse off. As a result of that new results emerge.

The rest of the paper is organised as follows. Section 2 presents the model. Section 5 discusses the main implications of pricing with customer recognition when consumers are either loyal to a specific degree to one firm or the other. Conclusions are presented in Section 8.

2. The model

Two firms, A and B, produce at zero marginal cost nondurable goods A and B.[14] There are two periods, 1 and 2. On the demand side, there is a large number of consumers, with mass normalized to one, each of whom wishes to buy a single unit of either good A or B in each of the two periods. As in Shilony (1977) each consumer is either loyal with degree $\gamma>0$ to firm A or B.[15] This means that consumers will buy from the most preferred firm as long as its price is not undercut by more than γ. As Raju, et al. (1990), γ can be used as a measure of the degree of a consumer's brand loyalty, defined as the minimum difference between the prices of the two competing brands necessary to induce consumers to buy the wrong brand.[16] Suppose also that half of consumers are loyal to firm A, while the remaining are loyal to firm B. It is also assumed that consumer preferences are fixed across periods.[17] Finally, consumers have a common reservation price v, which is sufficiently high so that the duopoly equilibrium exhibits competition. More precisely, $v > \left(2+\sqrt{2+2\delta^2-2\delta\sqrt{2}}\right)\gamma$.

2.1. Consumers' behaviour

In order to isolate the strategic effects of price discrimination with customer recognition it is assumed that consumers are myopic (or naive). This means that in period 1, consumers do not anticipate that the next period's prices may depend on their current behaviour. After firms have set their prices, consumers shop for the better bargain. The net utility of a loyal consumer to firm A purchasing good *A* at price p_A is $v-p_A$, while the net utility of purchasing good B at price p_B is $v-\gamma-p_B$. Thus, a loyal consumer to firm A buys good A whenever $p_A<p_B+\gamma$. Similarly, a loyal consumer to firm B buys good A whenever $p_A<p_B-\gamma$. Reversing the previous inequalities one gets the conditions under which consumers buy brand B.

2.2. Firms' behaviour

In each period firms act simultaneously and non-cooperatively. In the first period, consumers are *anonymous* and firms quote the same price for all consumers. In the second period, whether or not a consumer bought from the firm in the initial period may reveal that consumer's brand preference. When firms have the required information, they will set different prices to their own customers and to the rival's previous customers. When nothing is learned from the initial period, price discrimination is not feasible and firms again quote a single price to all consumers. Firms discount future profits using a common discount factor δ.

3. Benchmarks

Before proceeding, two static benchmarks are examined. The first, considers the case where consumers are non-anonymous and firms can engage in price discrimination. The other, considers the case where price discrimination, cannot occur either because firms have no information to recognise customers or because price discrimination is illegal.

[12] On the Pros and Cons of Price Discrimination, Chen (2005) argues also that firms tend to be worse off being able to recognise consumers and price discriminate. Targeted pricing is also bad for profits in Shaffer and Zhang (1995) and Bester and Petrakis (1996).

[13] There are however some models where firms can benefit from targeted pricing. This conclusion might be obtained when firms are asymmetric (e.g. Shaffer and Zhang (2000)), when firms targetability is imperfect and asymmetric (Chen et al. (2001)) and when only one of the two firms can recognise customers and price discriminate (Chen and Zhang (2009) and Esteves (2009a)).

[14] The assumption of zero marginal costs can be relaxed without altering the basic nature of the results derived throughout the model.

[15] Even though the paper considers that the market is segmented according to brand loyalty, the model also accommodates other interpretations such as search costs, transportation costs and switching costs.

[16] Raju, et al. (1990) present also the case with asymmetric loyalties towards the two firms.

[17] The assumption of fixed-preferences across periods is a requirement in models where firms can price discriminate based on past behaviour. If preferences were not fixed from period to period, the knowledge of a consumer's first period choice would provide no information about his second period preferences, so firms could not engage in price discrimination. For an analysis where preferences change over time, see for instance, Caminal and Matutes (1990) and Fudenberg and Tirole (2000, section 6).

3.1. Customer recognition and price discrimination

As in Thisse and Vives (1988), suppose that firms can observe each consumer brand preference and price accordingly. (Here, the main difference is that there are only two types of consumers.) Let p_{ii} denote the price charged by firm i to consumers who prefer brand i and p_{ij} the price charged by firm i to consumers who prefer brand j. The lowest price say firm B can offer to a consumer which prefers brand A is the marginal cost price, which in this case is equal to zero. But then, in order to prevent the consumer from being tempted by the rival's price, firm A needs to offer a price generating the same level of utility. It is straightforward to show that in equilibrium the price firm B quotes to consumers loyal to A is $p_{BA}=0$ and so p_{AA} must satisfy $v-p_{AA}=v-\gamma$, from which we obtain $p_{AA}=\gamma$. In sum, when firms are able to recognise customers and price discrimination is allowed, Bertrand competition in each segment of customers leads equilibrium prices to:

$$p_{ii}=\gamma \text{ and } p_{ij}=0, \text{ for } i,j=A,B \text{ and } i\neq j. \tag{1}$$

Equilibrium profit per firm when it recognises customers and prices accordingly, denoted π_D, is given by:

$$\pi_D=\frac{1}{2}\gamma. \tag{2}$$

3.2. No customer recognition and no price discrimination

As said, here it is assumed that price discrimination cannot occur. This benchmark is the special case of Shilony's (1977) model when there are only two firms in the market. Following Shilony a pure strategy equilibrium in prices fails to exist.[18] The intuition for the inexistence of a pure strategy price equilibrium is that although each firm can always guarantee itself a profit equal to $\frac{1}{2}\gamma$ by selling at price γ exclusively to its loyal segment, the presence of a positive fraction of disloyal customers creates a tension between the firm's incentives to price low in order to attract this latter set of customers and to price high as a way to extract rents from its loyal customers. Therefore, each firm follows a mixed pricing strategy as an attempt to prevent the rival from systematically predicting its price, which in turn makes undercutting less likely.[19]

Suppose that firm i selects a price randomly from the cdf $F_i(p)$. In a symmetric mixed strategy equilibrium, both firms follow the same pricing strategy, thus, for the sake of simplicity write $F_i(p)=F_j(p)=F(p)$. Suppose further that the support of the equilibrium prices is $[p_{\min}, p_{\max}]$. When firm i chooses any price that belongs to the equilibrium support of prices, and firm j uses the cdf $F(p)$, firm i's expected profit is always equal to a constant, which is denoted K. Since a firm can always guarantee itself a profit equal to $\frac{1}{2}\gamma$, it immediately follows that $K\geq\frac{1}{2}\gamma$. When firm i charges price p, two events are relevant. Firstly, p is the lowest price so all consumers go to firm i. This event occurs with probability $[1-F(p+\gamma)]$. Secondly, p is such that both firms share the market. This event occurs with probability $[F(p+\gamma)-F(p-\gamma)]$. The case where p is so high that it precludes firm i from getting any demand is not relevant because in that specific case firm i realises no profit. Hence, firm i's expected profit, denoted $E\pi(p)$, is

$$E\pi(p)=p[1-F(p+\gamma)]+\frac{1}{2}p[F(p+\gamma)-F(p-\gamma)].$$

In equilibrium, expected profit satisfies:

$$p\left(1-\frac{1}{2}F(p+\gamma)-\frac{1}{2}F(p-\gamma)\right)=K. \tag{3}$$

Since this benchmark case is an application of Shilony (1977) it is straightforward to obtain the equilibrium strategies and payoffs.[20]

Proposition 1. *Whenever firms cannot engage in price discrimination the Nash equilibrium is as follows:*

(i) *Each firm chooses a price randomly from the nondegenerate distribution function*

$$F(p)=\begin{Bmatrix} 0 & \text{if} & p\leq p_{\min} \\ 1-\dfrac{p_{\min}+\gamma}{(p+\gamma)} & \text{if} & p_{\min}\leq p\leq p_{\min}+\gamma \\ 2-\dfrac{p_{\min}+\gamma}{(p-\gamma)} & \text{if} & p_{\min}+\gamma\leq p\leq p_{\max} \\ 1 & \text{if} & p\geq p_{\max} \end{Bmatrix}, \tag{4}$$

The minimum and maximum equilibrium prices are respectively equal to

$$p_{\min}=\sqrt{2}\gamma, \tag{5}$$

and

$$p_{\max}=\left(2+\sqrt{2}\right)\gamma. \tag{6}$$

Because prices are bounded, i.e. $p_{\max}\leqq v$ *it follows that* $\left(2+\sqrt{2}\right)\gamma\leqq v$, *which is true under the model assumptions.*

(ii) *Each firm expected profit is in equilibrium equal to*

$$E\pi_{ND}=K=\frac{1}{2}\left(1+\sqrt{2}\right)\gamma. \tag{7}$$

As in Shilony because $K>\frac{1}{2}\gamma$ firms can make higher profits when they use mixed strategies and stay unpredictable.

Corollary 1. *With no discrimination the variance in prices is equal to* $Var_{ND}(p)=x_{ND}\gamma^2$, *where* x_{ND} *is a constant.*

Proof. See the Appendix A.

In the appendix it is given the exact expression for x_{ND}. It is also possible to verify that the variance of prices is approximately equal to $0.275\gamma^2$. This result predicts that price dispersion, measured by the variance of prices, tends to be greater in product-markets where customers are more loyal, and lends support to recent empirical research showing that observed price dispersion is mainly due to perceived differences among retailers related to branding, trust and awareness (e.g. Brynjolfsson and Smith (2000a)).[21]

From the two benchmark cases analysed it is possible to establish the following proposition.

[18] On the one hand, the marginal cost price cannot be an equilibrium of this game since by charging price γ each firm can always guarantee itself a profit equal to $\frac{1}{2}\gamma$. On the other hand, when both firms set price v, it is always profitable for a given firm to slightly decrease its price to $v-\varepsilon$ and capture the remaining customers. Any price lower than v but greater than or equal to $v-\gamma$ is dominated by v as it would give firm i the same demand and lower profits. However, if firm i chooses to undercut its rival charging a price $(v-\gamma-\varepsilon)$, it would capture all consumers and its profits would be equal to $(v-\gamma-\varepsilon)$. This deviation is profitable for firm i as long as v is high enough, i.e. $v>2\gamma$. In other words, (v,v) is a pure strategy equilibrium if $\gamma>\frac{v}{2}$. However, this condition contradicts the assumption $v>\left(2+\sqrt{2+2\delta^2-2\delta\sqrt{2}}\right)\gamma$.

[19] In a recent empirical paper, Baye et al. (2004) provide evidence that in internet markets those firms that adopted predictable pricing strategies were driven out of the market. For that reason, they claim that unpredictability in prices—i.e. setting prices at random—is widely used in those markets and is an effective way of avoiding aggressive price competition in online markets.

[20] The interested reader can find a formal proof in Shilony (1977) or from the author upon request.

[21] Although γ has been used as a brand loyalty parameter, it may equally be interpreted as a switching cost or as a cost of acquiring information. In other words, if customers were perfectly informed about their most preferred firm but would have to bear a cost to obtain information from other firms (due to the existence of search costs) our model would predict that the level of price dispersion would be greater in markets where it is more difficult to acquire such information.

Proposition 2.

(i) *Firms are better off when they cannot engage in price discrimination; either because customers are anonymous or because price discrimination is illegal.*
(ii) *Consumers pay strictly lower prices under discrimination than under no-discrimination.*

4. Pricing with customer recognition

Throughout this section it is assumed that, where feasible, price discrimination is permitted. Price discrimination can only occur in the second period if firms learn the consumers' preferences from the initial period. We will see that when both firms set (approximately) the same price in period 1, they will share equally the market, and consumer tastes are fully revealed. On the other hand, if firms set significantly different prices in period 1, one firm attracts all consumers and nothing is learned by period 2.

In period 1 each consumer type is a "mystery", so each firm sets a single first period price, denoted p_i^1. In period 2, if price discrimination is feasible, each firm quotes a different price to loyal and disloyal customers (those that bought previously from the rival). Let p_{iL}^2 and p_{iD}^2 denote firm i's second-period prices to loyal and disloyal customers, respectively. If nothing is learned from period 1, both firms charge again a single price in period 2, denoted p_i^2. In each period firms set prices simultaneously. Next, to derive the subgame perfect equilibrium, the game is solved working backwards from the second period.

4.1. Second-period pricing

Depending on first period prices (and correspondent market shares), two scenarios are possible in period 2. In one scenario, firms share the market in period 1, i.e. $(D_i, D_j) = \left(\frac{1}{2}, \frac{1}{2}\right)$, $i,j = A, B$, and both learn the consumer tastes in the subsequent period. In the other, the first period market is entirely served by the same firm, i.e. $(D_i, D_j) = (0, 1)$, $i,j = A, B$, and nothing is learned about consumer tastes in the second period.

4.1.1. Subgame 1: Both firms have information to price discriminate

When p^1 is such that firms share the market equally in period 1, consumer tastes are fully revealed in the subsequent period. Let $G_i(p^1)$ denote firm i's cdf of first period prices in the overall game. In a symmetric equilibrium $G_i(p^1) = G_j(p^1) = G(p^1)$. For a given price p^1 chosen by firm i in period 1, assuming that firm j sets its price according to G, both firms share the first period market with probability $[G(p^1+\gamma) - G(p^1-\gamma)]$ and in period 2 both firms have the required information to price discriminate. This subgame is identical to the benchmark case with non-anonymous consumers and price discrimination. Hence, under symmetry, each firm's equilibrium price to loyal and disloyal customers is respectively given by

$$p_L^2 = \gamma \text{ and } p_D^2 = 0, \tag{8}$$

and the second-period profit with discrimination, denoted π_D^2, is

$$\pi_D^2 = \frac{1}{2}\gamma. \tag{9}$$

Note that when first-period prices fully reveal consumer preferences, price discrimination intensifies competition in each segment, leading all prices to fall. According to Corts (1998), this all-out competition result is common in models where the market exhibits best-response asymmetry.[22] A special feature of this model is that even though firms offer lower second-period prices to their rival's customers (indeed, they quote the marginal cost price) in order to entice them to switch, there is no switching in equilibrium and all consumers buy efficiently.[23,24]

4.1.2. Subgame 2: Firms learn nothing and price discrimination is unfeasible

In this case all consumers buy from the same firm in period 1. With no loss of generality, take the behaviour of firm A in the first period. If firm A's price p^1 is much lower than the rival's price, all consumers go to firm A. This happens with probability $[1 - G(p^1+\gamma)]$. Otherwise, with probability $G(p^1+\gamma)$ firm A's first period price is much higher than firm B's price and no consumer goes to firm A. In either case, both firms learn nothing in the second period. Price discrimination cannot occur and the second-period pricing is a replication of the benchmark with no-discrimination. Let $E(\pi_{NR}^2)$ denote the second period expected equilibrium profit with non recognition of customers. From Eq. (7) it follows that:

$$E\left(\pi_{NR}^2\right) = \frac{1}{2}\left(1 + \sqrt{2}\right)\gamma. \tag{10}$$

Summing up, the second period expected profit per firm is

$$E\pi^2 = \left[G\left(p^1+\gamma\right) - G\left(p^1-\gamma\right)\right]\pi_D^2 + \left\{1 - \left[G\left(p^1+\gamma\right) - G\left(p^1-\gamma\right)\right]\right\}E\left(\pi_{NR}^2\right). \tag{11}$$

4.2. First-period pricing

Consider next the choice of first-period prices. Because firms are forward looking they take today's price decisions rationally anticipating *how* they will affect their subsequent profit. Note that firms take into account that their choice of first period prices will determine their knowledge of consumer tastes in the second period and thus the feasibility of price discrimination.

As in the benchmark case with anonymous consumers, it is straightforward to show that in the first-period of the repeated game there is no equilibrium in pure strategies. There exists rather a mixed strategy equilibrium. I prove the existence of such an equilibrium by construction. As previously discussed, when firm i charges $p^1 \in [p_{\min}^1, p_{\max}^1]$, it captures the entire market if $p_j^1 > p^1 + \gamma$. This happens with probability $[1 - G(p^1+\gamma)]$, and yields a profit equal to p^1. It sells nothing when $p_j^1 < p^1 - \gamma$. This occurs with probability $G(p^1-\gamma)$ and yields a zero economic profit. Finally, it sells exclusively to its loyal segment if $p^1 - \gamma < p_j^1 < p^1 + \gamma$, which in turn happens with probability $[G(p^1+\gamma) - G(p^1-\gamma)]$ and yields a profit equal to $\frac{1}{2}p^1$. Thus, each firm's first period expected profit is

$$E\pi^1 = p^1\left[1 - G\left(p^1+\gamma\right)\right] + \frac{1}{2}p^1\left[G\left(p^1+\gamma\right) - G\left(p^1-\gamma\right)\right]. \tag{12}$$

Overall expected profit is:

$$E\pi = E\pi^1 + \delta\left[G\left(p^1+\gamma\right) - G\left(p^1-\gamma\right)\right]\left[\pi_D^2 - E\left(\pi_{NR}^2\right)\right] + \delta E\left(\pi_{NR}^2\right) \tag{13}$$

For the sake of simplicity, suppose that the overall expected profit is equal to a constant C. In a mixed strategy equilibrium, any price

[22] It is important to stress that this all-out competition result requires that both firms can engage in price discrimination. When one firm has information to use price discrimination while the other does not it may happen that some prices rise (see Esteves (2009a)).

[23] This no poaching result is a consequence of the two-point distribution of demand, and would not extend to a discrete model with more than two types.

[24] The same result arises in Thisse and Vives (1988).

chosen from a firm's price support should generate the same expected profit, thus using Eqs. (9), (10) and (12) it follows that:

$$C = p^1\left[1-G\left(p^1+\gamma\right)\right]+\delta\frac{1}{2}\gamma\left(1+\sqrt{2}\right) + \frac{1}{2}\left[G\left(p^1+\gamma\right)-G\left(p^1-\gamma\right)\right]\left(p^1-\delta\sqrt{2\gamma}\right). \tag{14}$$

The next proposition provides a complete characterisation of the subgame perfect nash equilibrium of this game.

Proposition 3. *There is a symmetric mixed strategy subgame perfect Nash equilibrium in which:*

(i) *each firm's first-period price is randomly chosen from the cdf*

$$G\left(p^1\right)=\begin{Bmatrix} 0 & \text{if} \quad p^1\leq p^1_{\min} \\ 1-\dfrac{p^1_{\min}+\left(1-\delta\sqrt{2}\right)\gamma}{p^1+\left(1-\delta\sqrt{2}\right)\gamma} & \text{if} \quad p^1_{\min}\leq p^1\leq p^1_{\min}+\gamma \\ 2-\dfrac{p^1_{\min}+\left(1+\delta\sqrt{2}\right)\gamma}{p^1-\left(1-\delta\sqrt{2}\right)\gamma} & \text{if} \quad p^1_{\min}+\gamma\leq p^1\leq p^1_{\max} \\ 1 & \text{if} \quad p^1\geq p^1_{\max}\end{Bmatrix} \tag{15}$$

where the minimum and maximum equilibrium prices are respectively given by

$$p^1_{min}=\left(\sqrt{2+2\delta^2-2\delta\sqrt{2}}\right)\gamma \tag{16}$$

and

$$p^1_{max}=\left(2+\sqrt{2+2\delta^2-2\delta\sqrt{2}}\right)\gamma. \tag{17}$$

Because prices are bounded, $p^1_{\max}\leqq v$*, implies that* $\left(2+\sqrt{2+2\delta^2-2\delta\sqrt{2}}\right)\gamma\leqq v$ *(which satisfies the model assumptions).*

(ii) *Each firm earns an expected overall equilibrium profit equal to*

$$E\pi = C = \frac{1}{2}\left((1+\delta)+\sqrt{\left(2+2\delta^2-2\delta\sqrt{2}\right)}\right)\gamma. \tag{18}$$

Proof. See the Appendix A.

Corollary 2. *When price discrimination is introduced and* $\delta=1$*, the variance of prices is* $Var_D(p)=x_D\gamma^2$*, where* x_D *is a constant.*

Proof. See the Appendix A.

The constant x_D is obtained in the appendix. It is possible to see that x_D is approximately equal to 0.3641. Corollary 2 shows that when firms are allowed to price discriminate, first period prices remain dispersed. In fact, price dispersion only disappears when consumers are indifferent between both firms (i.e. when $\gamma=0$). Comparing corollary 1 and 2, one can see that $x_D>x_{ND}$ thus the level of first period price dispersion with discrimination is greater than if discrimination were banned. This suggests that although firms' heterogeneities (e.g. brand loyalty) are in fact an important force behind observed price dispersion, some other factors may also account for some fraction of price dispersion.[25]

[25] Baye et al. (2006a,b), note that despite observed price dispersion is mainly due to retailers heterogeneities (i.e. branding, reputation and trust), 28% of the observed dispersion is not explained by these heterogeneities.

5. Competitive effects of price discrimination

This section investigates the competitive effects of price discrimination. Do firms have an incentive to distort their first-period behaviour? Will they set higher or lower initial prices in comparison to the non-discrimination case? What is the impact of price discrimination on each firm's first-period demand? Do firms have an incentive to avoid learning and price discrimination?

5.1. First-period demand

Consider first the impact of price discrimination on first-period demand. Here I investigate whether a firm might have an incentive to *forgo* a positive demand in the initial period as an effective way to eschew learning and subsequent price discrimination.

Proposition 4. *The probability of first period demand sharing, given by* $q\in[0,1]$ *is equal to*

$$q = 1-2\frac{\left((p_{\min}+\gamma)^2-2(\delta\gamma)^2\right)}{\left(-1+2\delta\sqrt{2}\right)^2\gamma^2}\ln\left(\frac{(p_{\min}+\gamma)^2-2(\delta\gamma)^2}{\left(p_{\min}+2\gamma-\delta\sqrt{2}\gamma\right)\left(p_{\min}+\delta\sqrt{2}\gamma\right)}\right) -2\frac{\delta\sqrt{2}\gamma(2p_{\min}+\gamma)+p_{\min}(p_{\min}+\gamma)+2\delta^2\gamma^2}{\left(-1+2\delta\sqrt{2}\right)\left(p_{\min}+\delta\sqrt{2}\gamma\right)\left(p_{\min}+\gamma+\delta\sqrt{2}\gamma\right)}.$$

Proof. See the Appendix A.

Corollary 3. *When price discrimination is permitted forward-looking firms reduce the probability of sharing the first period market.*

Proof. See the Appendix A.

In the appendix provided we can see that when $\delta=0$ firms share the first period market with probability approximately equal to 0.81, while when $\delta=1$ this probability is equal to approximately 0.71. Fig. 1 plots the probability of first period demand sharing as a function of δ. We can see that the probability of both firms have positive first-period sales decreases as they become more patient.

An important result of this paper is that firms may strategically forgo any previous positive market share as an effective "weapon" to eschew the negative effects of price discrimination in subsequent periods. When firms foresee that they do better in period 2 when neither of them is able to learn the consumers' preferences, they may have an incentive to avoid learning, which in turn can be achieved by not sharing the initial market. Thus, forward-looking firms share the initial market less frequently than if price discrimination were not permitted.

In their surveys on BBPD, Fudenberg and Villas-Boas (2006) and Armstrong (2006) point also out that in the Fudenberg and Tirole's model, firms would do better in the second period with less symmetric market shares in period 1.[26,27]

5.2. First-period prices

Consider next the impact on first-period prices.

Proposition 5. *When firms are forward looking, first-period prices are below the static or no-discrimination levels.*

[26] In footnote 72 Armstrong notes that this is especially clear in chapter 3 of Esteves (2004) which is a previous version of the present paper.

[27] It is important to stress that in FT model with uniformly distributed preferences, if consumers are myopic, firms do not distort their first-period choices.

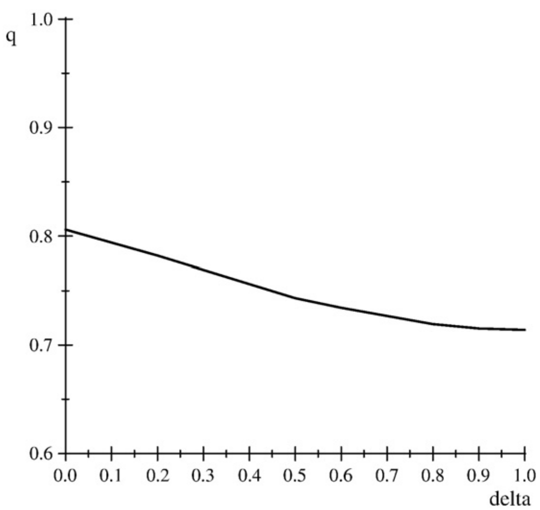

Fig. 1. Probability of demand sharing as a function of δ.

Comparing Eqs. (5) and (6) respectively with (16) and (17) it is easy to see that for $\delta \in [0, 1]$ the support of equilibrium prices falls.[28] In particular, if $\delta = 1$, the support of current prices falls from $\left[\sqrt{2}\gamma, \left(2+\sqrt{2}\right)\gamma\right]$ in the no-discrimination benchmark case to $\left[\left(\sqrt{4-2\sqrt{2}}\right)\gamma, \left(2+\sqrt{4-2\sqrt{2}}\right)\gamma\right]$ when there is price discrimination. Likewise, comparing the equilibrium distribution functions with and without discrimination, respectively given by $G(\cdot)$ and $F(\cdot)$, one finds that, F first-order stochastically dominates G. From the first-order stochastic dominance of F over G it follows that the average first-period price under discrimination is below its static counterpart. In fact, while under non-discrimination the average price is approximately equal to 2.542γ (see the proof of corollary 1 in the Appendix A), under discrimination the average first period price is approximately equal to 1.87γ (see the proof of corollary 2 in the Appendix A).

Fig. 2 is plotted for $\delta = 1$ and $\gamma = 1$. It shows also that when price discrimination is permitted, firms charge low first period prices more frequently than high prices. This result is in contrast with Fudenberg and Tirole's conclusion that the permission of price discrimination leads firms to *raise* (or at least to not reduce) their first period prices. Specifically, if consumers are forward looking they anticipate lower future prices due to poaching and become less price sensitive in the initial period. As a result, firms can choose higher first period prices than if poaching were not permitted. Surprisingly, Fudenberg and Tirole show that if consumers are myopic and their tastes are uniformly distributed, firms do not distort their first-period prices. In contrast to FT model, Proposition 5 shows first that forward-looking firms do in fact distort their first period pricing behaviour even when consumers are myopic. Second, it shows that first-period prices when price discrimination can occur tend to be below the static levels as in the switching cost approach. Nevertheless, the intuition behind Proposition 5 is quite different from that in the switching cost approach. In Chen (1997), the presence of switching costs allows firms to lock-in their old customers and to offer new customers a lower price, in order to entice them to switch from the rival. Thus, firms have an incentive to increase first-period market share as a way to increase their base of locked-in customers. This explains why in Chen's model firms price below static levels in the first period and then raise their prices once consumers are locked-in. Here firms price below the static levels because they try to avoid learning and subsequent price discrimination which will be achieved by not sharing the market in period 1. To attain that goal a firm needs to set its price far apart from the rival. However, because each firm prefers to be the leader, it may have an incentive to price more aggressively. This gives rise to lower first-period prices.

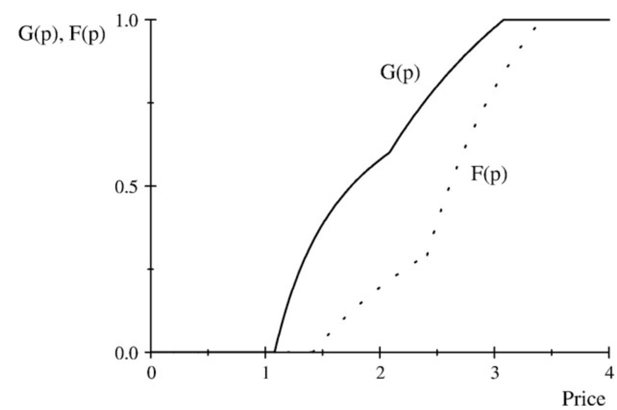

Fig. 2. Cumulative distribution functions for prices.

The results derived in the context of this simple model show that conclusions about the first-period effects of behaviour-based price discrimination and customer recognition might change when firms follow unpredictable pricing strategies (which tends to be the case in internet markets). Obviously, conclusions do depend on what is learned about consumers, and so on the distribution of consumer types.

5.3. Profits

Look next at the profitability of behaviour-based price discrimination and customer recognition in the present framework.

Proposition 6. *Price discrimination enabled by customer recognition hurts not only second period profit but also first period profit. Thus, overall expected profit decreases when firms learn the consumers types and become able to employ behaviour-based price discrimination.*

It was already shown that price discrimination is bad for second-period profits. To prove the second part of Proposition 6, note that if discrimination were not permitted, second-period expected profit would be a replication of first period profit under anonymous consumers/no discrimination. In this case, overall expected profit with no discrimination would be equal to $\frac{1}{2}(1+\delta)\left(1+\sqrt{2}\right)\gamma$. In contrast, when discrimination is allowed, overall expected profit is equal to $\frac{1}{2}\left((1+\delta)+\sqrt{\left(2+2\delta^2-2\delta\sqrt{2}\right)}\right)\gamma$. It is, then, straightforward to prove that, apart from the special case where $\delta = 0$, expected overall profit when discrimination is permitted is always below its non-discrimination counterpart.

Expected first period profit, $E\pi^1$, can be derived from $E\pi$ given by Eq. (13). Let q represent the probability of first period demand sharing, where $q \in [0, 1]$. Then,

$$E\pi^1 = E\pi - \delta E\left(\pi^2_{NR}\right) - \delta\left[G\left(p^1+\gamma\right) - G\left(p^1-\gamma\right)\right]\left[\pi^2_D - E\left(\pi^2_{NR}\right)\right]$$

from which it follows that

$$E\pi^1 - E\pi_{ND} = \frac{1}{2}\gamma\left[\sqrt{\left(2+2\delta^2-2\delta\sqrt{2}\right)} - (1+\delta(1-q))\left(\sqrt{2}\right)\right].$$

It is easy to see that for $0 < \delta \leqq 1$, the above difference is always negative and so first-period profit is clearly below its non-discrimination counterpart. This is due to (i) a smaller probability of positive market share in period 1 and (ii) lower first period prices.

[28] This conclusion is valid for $\delta \leq \sqrt{2}$.

6. Asymmetric equilibrium

As seen before, firms may have an incentive to avoid learning and price discrimination based on customer recognition by not sharing the initial market. So far the model predicts that there is a bias towards asymmetric outcomes in period 1. A natural question is whether firms can easily sustain the no learning result and the consequent high future prices and profits in a many-period model. The aim of this section is to investigate the circumstances under which an asymmetric pure strategy equilibrium may exist by which price discrimination is completely driven out of the market in subsequent periods. For a first approach to this question, and to simplify, consider that $\delta>1$ can be interpreted as a proxy for many future periods.

Proposition 7.

(i) *If* $\delta<\frac{3\sqrt{2}}{4}$ *there is no asymmetric pure strategy equilibrium.*

(ii) *If* $\delta\geqq\frac{3\sqrt{2}}{4}$ *an asymmetric pure strategy equilibrium exists in the first period with prices* (p_i^1, p_j^1) *such that firm i sets a low price and captures all the consumers whilst firm j gets nothing. For a given* p_i^1*, it must be the case that* $p_j^1>p_i^1+\gamma$*. The equilibrium is defined as follows:*

(a) *For* $\frac{3\sqrt{2}}{4}\leqq\delta<\sqrt{2}$ *and* $p_j^1>p_i^1+\gamma$*,*

$$p_i^1=\left(\sqrt{2}\delta-1\right)\gamma \tag{19}$$

and overall profits per firm are given by

$$\pi_i=\left[\frac{1}{2}\delta\left(3\sqrt{2}+1\right)-1\right]\gamma, \tag{20}$$

$$\pi_j=\frac{1}{2}\delta\left(1+\sqrt{2}\right)\gamma, \tag{21}$$

(b) *For* $\delta\geqq\sqrt{2}$ *and* $p_j^1>p_i^1+\gamma$*,*

$$p_i^1=\gamma,$$

and overall profits per firm are now given by

$$\pi_i=\left(1+\frac{1}{2}\delta\left(1+\sqrt{2}\right)\right)\gamma, \tag{22}$$

$$\pi_j=\left(\frac{1}{2}\delta\left(1+\sqrt{2}\right)\right)\gamma. \tag{23}$$

Proof. See the Appendix A.

Proposition 7 shows that even though firms are symmetric ex-ante, we may have an asymmetric first-period equilibrium in pure strategies where one firm quotes a low price and captures all consumers in the initial period and where the rival firm has no incentive to match this low-price firm because its profits will then be low in the second period. However, we expect that the asymmetric equilibrium can only be sustained in a many-period model. Otherwise, firms would not have an incentive to sacrifice current profits; and thereby, the equilibrium would be the one defined in proposition 3.

The intuition is that when firms value future profits more than current profits, they might be increasingly willing to sacrifice current profits as an effective way to fight against the negative effects of price discrimination. When firms foresee that by learning today, the discrimination game will be played for many repeated periods, they may have an incentive to completely avoid the negative effects of discrimination, and in doing so, one of the firms might even be willing to forgo a positive market share and profit in period 1. Although there is no explicit agreement between firms to restrict discrimination practices in subsequent periods, this simple example suggests that, under certain conditions, when one firm quotes a low enough price it will not be undercutted by its rival because the latter would not find it profitable to do so. This suggests that under certain circumstances *not* sharing the first period market is an effective "weapon" for avoiding the all-out competition result that would otherwise emerge in subsequent periods.[29]

It is well known that when profits are lower with discrimination, firms would become better off by colluding and committing not to discriminate. However, in the absence of any collective commitment, it is generally the case that even when one firm unilaterally commits to uniform pricing, the other firm finds it profitable to discriminate. In this regard, Thisse and Vives (1988) show that when one firm commits to uniform pricing, price discrimination is a dominant strategy for the other firm. The reason is that once one firm is committed to uniform pricing, the other firm is better off unconstrained in its pricing strategy. Thus, in this case, a unilateral commitment would not solve the prisoner's dilemma. One important implication of proposition 7 is that uniform pricing may arise without any collective action; it may arise on the basis of unilateral actions. Specifically, the choice of a low enough price by a firm could be interpreted as a credible kind of commitment to uniform pricing in the next stages of the game. For a high enough δ, the choice of a low price by a given firm would solve the prisoner's dilemma because at such a price, it would not be worth the other firm undercutting that price; and thus this latter firm would also be committed to uniform pricing in the subsequent period.

7. Welfare analysis

This section investigates the welfare effects of price discrimination through customer recognition in a market where consumers are loyal to a specific degree to one of the firms and where the firms follow in equilibrium a mixed pricing strategy. Given that price discrimination only occurs with some positive probability in the symmetric equilibrium analysed, the welfare analysis focuses only on this type of equilibrium. To simplify, throughout this section it is assumed that $\delta=1$. Total welfare can be written as v-"expected disutility cost". In the social optimal solution each consumer buys from the preferred firm, which happens exclusively when firms share the market. Otherwise, one group of consumers buy from the wrong firm thereby supporting a kind of disutility cost, i.e. γ.

Look first at the second-period welfare. Two situations are relevant. First, the case where price discrimination is permitted, and second, the case where there is no price discrimination. In the discrimination case, the second-period outcome is efficient because all consumers buy from the right firm. Second-period welfare is equal to

$$w_D^2=v. \tag{24}$$

Conversely, in the no-discrimination case, due to the randomized nature of the equilibrium, the outcome may not be fully efficient as some consumers may buy from the least preferred firm. Let q_N represent the probability of demand sharing when discrimination cannot occur. This means that under non-discrimination in period 2, all consumers buy efficiently with probability equal to q_N. With

[29] Zhang (2008) notes also that with customer recognition and product design, under some parameter values, we may have asymmetric first-period equilibria where one firms serves the entire market in period 1. She shows that the nature of the equilibrium depends on the degree of firm and consumer patience. When firms are patient and consumers myopic, one firm sells to all consumers in period 1, thus effectively avoiding customization in period 2.

probability $1-q_N$, half of consumers buy inefficiently and the remaining buy efficiently. Second-period welfare is now equal to $w_{ND}^2 = v - EDC_{ND}$, where EDC_{ND} is the expected disutility cost supported by all consumers under no discrimination.

$$EDC_{ND} = \frac{1}{2}\gamma(1-q_{ND}).$$

Thus, it ensues that

$$w_{ND}^2 = v - \frac{1}{2}\gamma(1-q_{ND}) \tag{25}$$

Look next at first-period welfare. Let q_D represent the probability of first-period demand sharing when discrimination can occur. Due to the randomized nature of the first-period equilibrium, some consumers may buy from the wrong firm, thus first-period welfare with discrimination is

$$w_D^1 = v - \frac{1}{2}\gamma(1-q_D). \tag{26}$$

When discrimination cannot occur firms have no incentives to distort their first period behaviour, and so they do not reduce the probability of sharing the market in period 1, which in this case is equal to q_{ND}. Thus, first-period welfare with no discrimination is

$$w_{ND}^1 = v - \frac{1}{2}\gamma(1-q_{ND}). \tag{27}$$

We can now obtain the overall expected welfare when price discrimination is allowed, denoted W_D, where

$$W_D = w_D^1 + q_D w_D^2 + (1-q_D)w_{ND}^2.$$

Using Eqs. (24), (25) and (26) it follows that

$$W_D = 2v - \frac{1}{2}\gamma(1-q_D)(2-q_{ND}) \tag{28}$$

Overall welfare when discrimination is not permitted, i.e. W_{ND}, is now equal to $w_{ND}^1 + w_{ND}^2$. Using Eqs. (25) and (27) it follows that,

$$W_{ND} = 2v - \gamma(1-q_{ND}). \tag{29}$$

It is now possible to establish the following proposition.

Proposition 8. *Price discrimination based on customer recognition is bad for profits, but good for consumer surplus and welfare.*

Proof. See the Appendix A.

Although a formal proof is presented in the appendix provided note that q_{ND} is approximately equal to 0.81 and, for the case where $\delta=1$, q_D is approximately equal to 0.71. It ensues that $W_D - W_{ND} \simeq 0.01745\gamma$, $\Pi_D - \Pi_{ND} \simeq -1.746\gamma$ and $CS_D - CS_{ND} \simeq 1.7635\gamma$.

7.1. The value of recognition

Consider the following simple exercise. Compare the total price consumers are expected to pay when they buy anonymously with that they are expected to pay when firms can recognise them and price discriminate. The expected difference between these expected prices can be interpreted as the *value of recognition,* say *VR*, where

$$\begin{aligned} VR &= 2E(p)_{ND} - \left[E\left(p^1\right)_D + q(p_D) + (1-q)E(p)_{ND}\right] \\ &\simeq 2(2.54\gamma) - [1.87\gamma + q(0.5\gamma) + (1-q)(2.54\gamma)] \\ &\simeq (0.67 + 2.04q)\gamma. \end{aligned}$$

As $VR>0$ consumers are expected to pay less when firms do recognise them and price discriminate accordingly. If say, $\delta=1$, by behaving non-anonymously consumers are expected to save approximately 2.12γ.

Compare next the welfare results derived in this paper with those in the existing literature. Here, both the static and the first-period equilibrium of the repeated game with discrimination are in mixed strategies. This implies that some consumers may buy inefficiently. In contrast, when firms learn the consumers' tastes and employ behaviour-based price discrimination, consumers buy from their preferred firm in period 2, which is efficient. (As previously said, even though firms try to poach their rival's previous customers, there is no switch in equilibrium.)[30] Note also that when discrimination is introduced, it is more likely that in period 1 some consumers buy inefficiently, due to the reduction on the probability of demand sharing. However, as $W_D - W_{ND} > 0$ under discrimination the increase in efficiency in period 2 is greater than the decrease in efficiency in period 1. Particularly, price discrimination is welfare enhancing when it leads to more efficient shopping (i.e. if expected disutility cost falls with discrimination). In fact, welfare increases with discrimination because $EDC_D - EDC_{ND} < 0$ (i.e., $EDC_D - EDC_{ND}$ is approximately equal to -0.01745γ).

This paper highlights the importance of taking into account different forms of market competition when policy makers try to evaluate the welfare effects of price discrimination with customer recognition. In broad terms, when the equilibrium is in pure strategies as in Chen (1997) and Fudenberg and Tirole (2000) price discrimination is welfare reducing, due to excessive inefficient switching. In the Fudenberg–Tirole model, for instance, in period 1 all consumers buy from the closer firm, which is efficient. When firms recognise their own customers and their rival's customers, price discrimination leads some consumers to buy from the more distant firm in period 2, which is inefficient. As in these models without discrimination consumers always buy from the right firm, a ban on price discrimination would be socially desirable. Here, in contrast, because random pricing tends to generate some inefficient shopping, price discrimination can increase efficiency.

Although in Esteves (2009a) the first-period equilibrium is also in mixed strategies there is no inefficient shopping because firms offer an homogenous product and there are no switching costs. Thus, the unique source of social inefficiency, is the advertising intensity selected by firms, which endogenously determines the number of consumers that will enter the market. In this context it is shown that, at least when advertising costs are not too high, BBPD is generally good for profits (because only one of the firms discriminates), but bad for overall welfare and consumer surplus. In Chen and Zhang (2009) firms have a captive group of consumers and they only compete for the switchers. Efficiency considerations do not arise when all consumers have the same reservation price. They show however that price discrimination improves welfare when the reservation value of the switchers is less than the captives and BBPD allows the firms to separate captive from switchers and offer the switchers a lower price. When this gives rise to a market expansion effect, BBPD enhances social welfare.

The welfare results derived in this paper suggest that when the equilibrium market tends to generate random pricing, as is usually the case in internet markets, any attempt by firms to price discriminate based on customer recognition may be welfare enhancing provided that it leads to more efficient shopping. In this setting, public policy restricting the collection and use of consumers' private information would solve the industry prisoner's dilemma creating a mechanism in favour of uniform pricing rules and high industry profits. This would be a friendly competition world for firms looking for ways to boost

[30] It is important to stress that with more than two types price discrimination could also be welfare reducing due to inefficient switching.

their profits at the expense of consumer welfare. Thus, conclusions regarding the welfare implications of behaviour-based price discrimination and customer recognition do depend on the available information, which in turn depends on the form of consumer heterogeneity. Public policy in favour or against this form of price discrimination should be drawn up from a good economic understanding of each particular market.

8. Conclusions

This paper has studied the competitive effects of price discrimination based on customer recognition in a duopolistic market where the distribution of consumer types is discrete. The use of a discrete distribution for consumer tastes has raised issues not addressed in the literature so far. This is one of the first papers to analyse the implications of behaviour-based price discrimination in markets where firms choose random prices.

The present analysis has confirmed that more information leads to more intense competition and to a less favourable competitive outcome. As a result, it was shown that when firms foresee the negative effects of price discrimination they might have an incentive to distort their static behaviour. First, it was shown that firms may be willing to forgo a positive market share in period 1 as an effective way to eschew learning and price discrimination in the subsequent period. Specifically, it was proved that the probability of both firms having positive first period sales falls as they become more patient. When $\delta>1$ is used as a proxy for many future periods, it was shown that firms can completely avoid the drawbacks of price discrimination. More precisely, it was proved that in this case there is an asymmetric pure strategy equilibrium in period 1, in which one firm sets a low price and captures all consumers, but it is not worth the competitor matching this low-price firm as its profits would then be low in the second period. Second, it was shown that first period prices are below their static or non-discrimination counterparts. This latter result is quite the reverse of that achieved in the extant models where purchase history discloses information about brand preferences (e.g. Villas-Boas (1999) and Fudenberg and Tirole (2000)). Further, the intuition behind lower first period prices is quite different from that behind lower first period prices in the switching cost approach.

Although this model is at best a crude approximation of real markets, the stylised model addressed and the results derived herein suggest that conclusions regarding the economic and welfare effects of behaviour-based price discrimination and customer recognition do depend on what is learned about consumer demand, which in turn depends on the distribution of preferences. The welfare results obtained in this paper suggest that when the equilibrium market tends to generate random pricing, as is usually the case for instance in Internet markets, any attempt by firms to price discriminate based on customer recognition may be welfare enhancing provided that it leads to more efficient shopping. Thus, in those markets that could be reasonably well represented by the features of the current model it seems that it is the consumers and not the firms that will benefit the most from price discrimination enabled by customer recognition.

Appendix A

Proof of Corollary 1. The average price consumers expect to pay in the static or no-discrimination game is given by

$$\begin{aligned} E(p) &= \int_{sup} pf(p)dp \\ &= \left(1+\sqrt{2}\right)\gamma\left(\int_{(\sqrt{2})\gamma}^{(1+\sqrt{2})\gamma}\frac{p}{(p+\gamma)^2}dp + \int_{(1+\sqrt{2})\gamma}^{(2+\sqrt{2})\gamma}\frac{p}{(p-\gamma)^2}dp\right) \\ &= \left(1+\sqrt{2}\right)\gamma\left(\ln\left(\sqrt{2}+2\right)-\frac{1}{2}\ln 2-2\sqrt{2}+3\right) \end{aligned}$$

Being the variance of prices given by $\mathrm{Var}(p)=E(p^2)-[E(p)]^2$, where

$$\begin{aligned} E\left(p^2\right) &= \int_{\sqrt{2}\gamma}^{(2+\sqrt{2})\gamma} p^2 f(p)dp \\ &= \left(1+\sqrt{2}\right)\gamma\left(\int_{(\sqrt{2})\gamma}^{(1+\sqrt{2})\gamma}\frac{p^2}{(p+\gamma)^2}dp + \int_{(1+\sqrt{2})\gamma}^{(2+\sqrt{2})\gamma}\frac{p^2}{(p-\gamma)^2}dp\right) \\ &= \gamma^2\left(\sqrt{2}+1\right)\left(4\ln\left(\sqrt{2}+1\right)-2\ln\left(\sqrt{2}+2\right)-\ln 2+\sqrt{2}+1\right) \end{aligned}$$

it follows that

$$\begin{aligned} \mathrm{Var}(p) = E\left(p^2\right)-[E(p)]^2 = \gamma^2\left(\sqrt{2}+1\right)&\left(\ln\frac{1}{2}\frac{\left(\sqrt{2}+1\right)^4}{\left(\sqrt{2}+2\right)^2}\right. \\ &\left.-\left(\sqrt{2}+1\right)\left(\ln\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\right)-2\sqrt{2}+3\right)^2+\sqrt{2}+1\right) \end{aligned}$$

Using the fact that $x_{\mathrm{ND}} = \left(\sqrt{2}+1\right)\left(\ln\frac{1}{2}\frac{\left(\sqrt{2}+1\right)^4}{\left(\sqrt{2}+2\right)^2}-\left(\sqrt{2}+1\right)\times\left(\ln\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\right)-2\sqrt{2}+3\right)^2+\sqrt{2}+1\right)$ it follows that

$$\mathrm{Var}_{\mathrm{ND}}(p) = x_{\mathrm{ND}}\gamma^2.$$

It is straightforward to see that x_{ND} is approximately equal 0.275. *Q.E.D.*

Proof of Proposition 3. In a mixed strategy equilibrium for any $p\in[p_{\min}, p_{\max}]$, overall expected profit is equal to:

$$C=p[1-G(p+\gamma)]+\delta\frac{1}{2}\gamma\left(1+\sqrt{2}\right)+\frac{1}{2}[G(p+\gamma)-G(p-\gamma)]\left(p-\delta\sqrt{2}\gamma\right). \quad (30)$$

Suppose that

p_1 is such that $p_1-\gamma = p_{\min}$
p_2 is such that $p_2+\gamma = p_{max}$

Then,

$$\forall p\le p_1, G(p-\gamma) = 0$$

$$\forall p\ge p_2, G(p+\gamma) = 1.$$

Using Eq. (30) it follows that

$$p_1[1-G(p_1+\gamma)]+\delta\frac{1}{2}\gamma\left(1+\sqrt{2}\right)+\frac{1}{2}G(p_1+\gamma)\left(p_1-\delta\sqrt{2}\gamma\right) = C \quad (31)$$

and

$$\delta\frac{1}{2}\gamma\left(1+\sqrt{2}\right)+\frac{1}{2}[1-G(p_2-\gamma)]\left(p_2-\delta\sqrt{2}\gamma\right) = C \quad (32)$$

Thus,

$$\forall p\le p_1\Rightarrow G(p+\gamma) = 2-\frac{2C-\delta\left(1-\sqrt{2}\right)\gamma}{p+\sqrt{2}\gamma\delta} \text{ or}$$

$$\forall p\le p_1\Rightarrow G(p) = 2-\frac{2C-\delta\left(1-\sqrt{2}\right)\gamma}{p-\left(1-\sqrt{2}\delta\right)\gamma}.$$

Similarly,

$$\forall p \geq p_2 \Rightarrow G(p-\gamma) = 1-\frac{2C-\delta\left(1+\sqrt{2}\right)\gamma}{p-\delta\sqrt{2}\gamma} \text{ or}$$
$$\forall p \geq p_2 \Rightarrow G(p) = 1-\frac{2C-\delta\left(1+\sqrt{2}\right)\gamma}{p+\gamma\left(1-\delta\sqrt{2}\right)}. \tag{33}$$

I now show that $p_1 = p_2$. Suppose first that $p_2 < p_1$. Then, $\forall p \in [p_2, p_1]$ it follows $G(p-\gamma)=0$ and $G(p+\gamma)=1$ thus

$$C = \delta\frac{1}{2}\gamma\left(1+\sqrt{2}\right) + \frac{1}{2}\left(p-\delta\sqrt{2}\gamma\right). \text{ A contradiction.}$$

Assume now that $p_2 > p_1$ and take $p \in [p_1, p_2]$ such that Eq. (30) holds.

$\exists \hat{p}$ s.t. $\hat{p}-\gamma = p_L < p_1$

$\hat{p}+\gamma = p_H > p_2$

Since $p_L < p_1$ and $p_H > p_2$, it follows that $G(\hat{p}) = 2-\frac{2C-\delta\left(1-\sqrt{2}\right)\gamma}{\hat{p}-\left(1-\sqrt{2}\delta\right)\gamma}$ and $G(\hat{p}) = 1-\frac{2C-\delta\left(1+\sqrt{2}\right)\gamma}{\hat{p}+\gamma\left(1-\delta\sqrt{2}\right)}$. From the continuity of G it must be true that $2-\frac{2C-\delta\left(1-\sqrt{2}\right)\gamma}{\hat{p}-\left(1-\sqrt{2}\delta\right)\gamma} = 1-\frac{2C-\delta\left(1+\sqrt{2}\right)\gamma}{\hat{p}+\gamma\left(1-\delta\sqrt{2}\right)}$, from which we obtain that $\hat{p} = \sqrt{2\gamma^2\delta^2-\gamma\left(\sqrt{2}\delta-1\right)\left(4C+\gamma-2\gamma\delta-\sqrt{2}\gamma\delta\right)}-\sqrt{2}\gamma\delta$. Since this must hold $\forall p \in [p_1, p_2]$ and they cannot all be equal it must be the case that $p_1 = p_2$. Since $p_1 = p_{\min}+\gamma$ and $p_2 = p_{\max}-\gamma$ it follows that $p_{\min}+\gamma = p_{\max}-\gamma$ or equivalently $p_{\max}-p_{\min} = 2\gamma$.

Let p^1 be the first period price then given that $p^1_{\max}-p^1_{\min} = 2\gamma$, it follows that $G(p^1)$ is continuous with no flat range and increasing in p^1 whenever $C > \frac{1}{2}\delta\left(1+\sqrt{2}\right)\gamma$. It is given by

$$G\left(p^1\right) = \begin{Bmatrix} 0 & \text{if} \quad p^1 < p^1_{\min} \\ 1-\dfrac{2C-\delta\left(1+\sqrt{2}\right)\gamma}{p^1+\left(1-\delta\sqrt{2}\right)\gamma} & \text{if} \quad p^1_{\min} \leqq p^1 \leqq p^1_{\max}-\gamma \\ 2-\dfrac{2C-\delta\left(1-\sqrt{2}\right)\gamma}{p^1-\left(1-\delta\sqrt{2}\right)\gamma} & \text{if} \quad p^1_{\max}-\gamma \leqq p^1 < p^1_{\max} \\ 1 & \text{if} \quad p^1 \geqq p^1_{\max} \end{Bmatrix} \tag{34}$$

Next it is shown that the cdf G is increasing in p^1 whenever $C > \frac{1}{2}\delta\left(1+\sqrt{2}\right)\gamma$. When $p^1_{\min} \leqq p \leqq p^1_{\max}-\gamma$, $\frac{\partial G(p^1)}{\partial p^1} = \frac{2C-\delta\left(1+\sqrt{2}\right)\gamma}{\left(p^1+\left(1-\delta\sqrt{2}\right)\gamma\right)^2}$ is positive if and only if $2C > \delta\left(1+\sqrt{2}\right)\gamma$. Similarly, when $p^1_{\max}-\gamma \leqq p \leqq p^1_{\max}$, $\frac{\partial G(p^1)}{\partial p^1} = \frac{2C-\delta\left(1-\sqrt{2}\right)\gamma}{\left(p^1-\left(1-\delta\sqrt{2}\right)\gamma\right)^2}$ is positive if and only if $2C > \delta\left(1-\sqrt{2}\right)\gamma$. Both conditions are satisfied whenever $C > \frac{1}{2}\delta\left(1+\sqrt{2}\right)\gamma$.

From $G(p^1_{\min}) = 0$ and $G(p^1_{\max}) = 1$ it follows that

$$p^1_{\min} = 2C-(1+\delta)\gamma \tag{35}$$

and

$$p^1_{max} = 2C+(1-\delta)\gamma \text{ where } p^1_{max} \leqq v. \tag{36}$$

From the continuity of $G(p^1)$ at $p^1 = p^1_{\max}-\gamma$ it follows that

$$1-\frac{2C-\delta\left(1+\sqrt{2}\right)\gamma}{p_{1\max}-\gamma+\left(1-\delta\sqrt{2}\right)\gamma} = 2-\frac{2C-\delta\left(1-\sqrt{2}\right)\gamma}{p_{1\max}-\gamma-\left(1-\delta\sqrt{2}\right)\gamma}.$$

Using the fact that $p_{1\max}-\gamma = 2C-\delta\gamma$ it follows that

$$C = \frac{1}{2}\left((1+\delta)+\sqrt{\left(2+2\delta^2-2\delta\sqrt{2}\right)}\right)\gamma. \tag{37}$$

Q.E.D.

Proof of Corollary 2. Under price discrimination, the expected first period price, when $\delta = 1$ is given by:

$$E\left(p^1\right)_D = \int_{\sup} p^1 g\left(p^1\right)dp^1 = y_1\int_{p\min}^{p_{\min}+\gamma}\frac{p}{(p+H)^2}dp + y_2\int_{p_{\min}+\gamma}^{p_{\max}}\frac{p}{(p-H)^2}dp$$

As $p_{\min} = \left(\sqrt{4-2\sqrt{2}}\right)\gamma$; $p_{\max} = \left(2+\sqrt{4-2\sqrt{2}}\right)\gamma$; $y_1 = \left(\sqrt{2}\sqrt{2-\sqrt{2}}-\sqrt{2}+1\right)\gamma$; $y_2 = \left(\sqrt{2}\sqrt{2-\sqrt{2}}+\sqrt{2}+1\right)\gamma$ and $H = \left(1-\sqrt{2}\right)\gamma$ it follows that

$$E\left(p^1\right)_D = y_1\left[\frac{H}{p_{\min}+\gamma+H}-\frac{H}{p_{\min}+H}+\ln\left(\frac{p_{\min}+H}{p_{\min}+\gamma+H}\right)\right] + y_2\left[\frac{H}{p_{\min}+\gamma-H}-\frac{H}{p_{\max}-H}+\ln\left(\frac{p_{\max}-H}{p_{\min}+\gamma-H}\right)\right]$$

which is approximately equal to 1.8716γ.

The variance of prices is given by $\mathrm{Var}(p) = E(p^2)-[E(p)]^2$. It follows that

$$E\left(\left(p^1\right)^2\right)_D = \int_{\sup}\left(p^1\right)^2 g\left(p^1\right)dp^1 = y_1\int_{p\min}^{p_{\min}+\gamma}\frac{p^2}{(p+H)^2}dp + y_2\int_{p_{\min}+\gamma}^{p_{\max}}\frac{p^2}{(p-H)^2}dp$$

from which we obtain:

$$E\left(\left(p^1\right)^2\right)_D = y_1\left[\gamma+\frac{H^2}{p_{\min}+H}-\frac{H^2}{p_{\min}+\gamma+H}+2H\ln\left(\frac{p_{\min}+H}{p_{\min}+\gamma+H}\right)\right] + y_2\left[\gamma+\frac{H^2}{H-p_{\max}}-\frac{H^2}{H-p_{\min}-\gamma}+2H\ln\left(\frac{H-p_{\max}}{H-p_{\min}-\gamma}\right)\right]$$

which is approximately equal to $3.867\gamma^2$. Therefore,

$$\mathrm{Var}_D\left(p^1\right) = E\left(\left(p^1\right)^2\right)_D-\left[E\left(p^1\right)_D\right]^2 = x_D\gamma^2.$$

It is straightforward to see that x_D is approximately equal to $0.3641\gamma^2$. *Q.E.D.*

Proof of Proposition 4. Because the model is symmetric both firms have the same support of prices. Then if we let q represent the probability of first-period demand sharing we have that

$$q = 1-2\int_{p_{\min}+\gamma}^{p_{\max}}\left(\int_{p_{\min}}^{p_A-\gamma}f(p_B)dp_B\right)f(p_A)dp_A \tag{38}$$

from which it follows that:

$$q=1-2\frac{\left((p_{\min}+\gamma)^2-2(\delta\gamma)^2\right)}{\left(-1+2\delta\sqrt{2}\right)^2\gamma^2}\ln\left(\frac{(p_{\min}+\gamma)^2-2(\delta\gamma)^2}{\left(p_{\min}+2\gamma-\delta\sqrt{2}\gamma\right)\left(p_{\min}+\delta\sqrt{2}\gamma\right)}\right)$$
$$-2\frac{\delta\sqrt{2}\gamma(2p_{\min}+\gamma)+p_{\min}(p_{\min}+\gamma)+2\delta^2\gamma^2}{\left(-1+2\delta\sqrt{2}\right)\left(p_{\min}+\delta\sqrt{2}\gamma\right)\left(p_{\min}+\gamma+\delta\sqrt{2}\gamma\right)}$$

Q.E.D.

Proof of Corollary 3. When price discrimination is not permitted or when firms are myopic $\delta=0$ and $p_{\min}=\sqrt{2}\gamma$. Thus, substituting these values in the general expression for q we obtain:

$$q(\delta=0)=3-4\sqrt{2}\ln\left(\frac{1}{2}\sqrt{2}+\frac{1}{2}\right)-6\ln\left(\frac{1}{2}\sqrt{2}+\frac{1}{2}\right)$$

which is approximately equal to 0.81.

Similarly, when firms are forward looking and $\delta=1$, it follows that $p_{\min}=\left(\sqrt{4-2\sqrt{2}}\right)\gamma$. We now obtain that $q(\delta=1)$ is equal to:

$$\left(\sqrt{2-\sqrt{2}}\left(\tfrac{36}{49}\sqrt{2}+\tfrac{32}{49}\right)\right)\left(\frac{303}{62}+\ln\left(\frac{2\sqrt{2}\sqrt{2-\sqrt{2}}+2}{2\sqrt{2}\sqrt{2-\sqrt{2}}-2\sqrt{2}+3}\right)\right.$$
$$\left.-\sqrt{2-\sqrt{2}}\left(\tfrac{53}{31}\sqrt{2}-\tfrac{72}{31}\right)-\tfrac{107}{31}\sqrt{2}\right)+\left(\tfrac{22}{49}-\tfrac{12}{49}\sqrt{2}\right)$$
$$\times\left(\frac{303}{62}+\ln\left(\frac{2\sqrt{2}\sqrt{2-\sqrt{2}}+2}{2\sqrt{2}\sqrt{2-\sqrt{2}}-2\sqrt{2}+3}\right)-\sqrt{2-\sqrt{2}}\left(\tfrac{53}{31}\sqrt{2}-\tfrac{72}{31}\right)-\tfrac{107}{31}\sqrt{2}\right)$$

which is approximately equal to 0.71. *Q.E.D.*

Proof of Proposition 7. Suppose there is an asymmetric pure strategy equilibrium with prices (p_i^1, p_j^1) such that firm i serves the entire market in period 1 while firm j has no demand. For a given p_i^1, it must be the case that $p_j^1>p_i^1+\gamma$. Because the market is entirely served by the same firm in the initial period, both firms learn nothing by the second period. In this situation, both firms set their prices randomly as in the static case with anonymous consumers. Thus, overall equilibrium profits per firm are given by,

$$\pi_i=p_i^1+\delta\frac{1}{2}\left(1+\sqrt{2}\right)\gamma$$

and

$$\pi_j=\delta\frac{1}{2}\left(1+\sqrt{2}\right)\gamma.$$

This can only be an equilibrium if firm j has no incentive to deviate. When firm j deviates two possible situations may occur. First, when firm j sets a price such that $p_j^1=p_i^1+\gamma$ it shares the first-period market equally with firm i and its profit from deviation, say π_j^d, is

$$\pi_j^d=\frac{1}{2}\left(p_i^1+\gamma\right)+\delta\left(\frac{1}{2}\gamma\right).$$

Second, when firm j sets a price such that $p_j^1=p_i^1-\gamma-\varepsilon$ it captures the entire market. In this case its profit from deviation is

$$\pi_j^d=p_i^1-\gamma-\varepsilon+\delta\frac{1}{2}\left(1+\sqrt{2}\right)\gamma.$$

Summing up, firm j has no incentive to deviate as long as $\pi_j=\delta\frac{1}{2}\left(1+\sqrt{2}\right)\gamma\geq\pi_j^d$, i.e. if the two following conditions hold:

$$\delta\frac{1}{2}\left(1+\sqrt{2}\right)\gamma\geq\frac{1}{2}\left(p_i^1+\gamma\right)+\delta\left(\frac{1}{2}\gamma\right)$$

and

$$\delta\frac{1}{2}\left(1+\sqrt{2}\right)\gamma\geq p_i^1-\gamma-\varepsilon+\delta\frac{1}{2}\left(1+\sqrt{2}\right)\gamma.$$

Or, equivalently if,

$$p_i^1\leq\sqrt{2}\delta\gamma-\gamma \tag{39}$$

and

$$p_i^1\leq\gamma+\varepsilon\lesseqqgtr\gamma\ \text{for }\varepsilon\text{ are sufficiently small.} \tag{40}$$

Since the equilibrium price must be equal or above the marginal cost, $p_i^1\geq0$, this implies that if the asymmetric equilibrium exists it must be the case that $\delta\geq\frac{\sqrt{2}}{2}$. From Eqs. (39) and (40) it follows that $\sqrt{2}\delta\gamma-\gamma\leq\gamma$, if and only if $\delta\leq\sqrt{2}$. In sum, when $\frac{\sqrt{2}}{2}<\delta\leqq\sqrt{2}$ firm i's first period price is given by $p_i^1=\sqrt{2}\delta\gamma-\gamma$ whilst when $\delta>\sqrt{2}$ firm i's first period price is $p_i^1=\gamma$. Thus, when $\frac{\sqrt{2}}{2}<\delta\leqq\sqrt{2}$ overall equilibrium profits are:

$$\pi_i=\frac{1}{2}\delta\left(3\sqrt{2}+1\right)\gamma-\gamma$$
$$\pi_j=\delta\frac{1}{2}\left(1+\sqrt{2}\right)\gamma.$$

When $\delta>\sqrt{2}$, overall equilibrium profits are:

$$\pi_i=\frac{1}{2}\delta\left(1+\sqrt{2}\right)\gamma+\gamma$$
$$\pi_j=\frac{1}{2}\delta\left(1+\sqrt{2}\right)\gamma.$$

Finally, to finish the proof one needs to verify that firm i has also no incentive to increase its price and share the market with firm j. Given that p_i^1 is such that allows firm i to serve the entire market, firm i could increase its price by 2γ and share the market with firm j. Its profit from deviation would be equal to:

$$\pi_i^d=\frac{p_i^1+2\gamma}{2}+\delta\left(\frac{1}{2}\gamma\right).$$

This deviation would not be profitable if

$$\frac{p_i^1+2\gamma}{2}+\delta\left(\frac{1}{2}\gamma\right)\leq p_i^1+\frac{1}{2}\delta\left(1+\sqrt{2}\right)\gamma$$

or, if

$$p_i^1\geq2\gamma-\sqrt{2}\delta\gamma.$$

When $p_i^1=\sqrt{2}\delta\gamma-\gamma$ the previous condition is satisfied if and only if

$$\sqrt{2}\delta\gamma-\gamma\geq2\gamma-\sqrt{2}\delta\gamma\Rightarrow\delta\geqq\frac{3\sqrt{2}}{4}.$$

Otherwise, when $p_i^1 = \gamma$, the above condition is satisfied if and only if

$$\gamma > 2\gamma - \sqrt{2}\delta\gamma \Rightarrow \delta > \frac{\sqrt{2}}{2}$$

which for $\delta > \sqrt{2}$ is always true. *Q.E.D.*

Proof of Proposition 8. Look at the variation in industry profits, total welfare and consumer surplus when moving from no discrimination to discrimination. It follows that

$$W_D - W_{ND} = \frac{\gamma}{2}(q_D(2 - q_{ND}) - q_{ND}).$$

Note that $W_D - W_{ND} > 0$ as long as $q_D(2 - q_{ND}) - q_{ND} > 0$. Since $q_{ND} > q_D$ it follows that $q_D(2 - q_{ND}) > q_{ND} > q_D$ and so $q_D(2 - q_{ND}) > q_D$ implies that $2 - q_{ND} > 0$ which is always true. Thus $W_D - W_{ND} > 0$.

When $\delta = 1$ industry expected profit with no discrimination is equal to $\Pi_{ND} = 2\left(1 + \sqrt{2}\right)\gamma$. In contrast, when discrimination is allowed, industry expected profit is equal to $\Pi_D = \left(2 + \sqrt{2}\sqrt{2 - \sqrt{2}}\right)\gamma$. Thus

$$\begin{aligned}\Pi_D - \Pi_{ND} &= \left(2 + \sqrt{2}\sqrt{2 - \sqrt{2}}\right)\gamma - 2\left(1 + \sqrt{2}\right)\gamma \\ &= \sqrt{2}\left(\sqrt{2 - \sqrt{2}} - 2\right)\gamma < 0.\end{aligned}$$

Since $\Pi_D - \Pi_{ND} < 0$ and $W_D - W_{ND} > 0$ it must be the case that $CS_D - CS_{ND} > 0$. *Q.E.D.*

References

Armstrong, M., 2006. Recent developments in the economics of price discrimination. In: Blundell, R., Newey, W., Persson, T. (Eds.), *Advances in Economics and Econometrics: Theory and Applications: Ninth World Congress of the Econometric Society*. Cambridge University Press, Cambridge, UK.

Baye, M., Morgan, J., Sholten, P., 2004. Price dispersion in the small and in the large: evidence from an internet price comparison site. Journal of Industrial Economics 52, 463–496.

Baye, M., Morgan, J., Sholten, P., 2006a. Persistent price dispersion in online markets. In: Jansen, D.W. (Ed.), *The New Economy and Beyond: Past, Present and Future*. Edward Elgar.

Baye, M., Morgan, J., Sholten, P., 2006b. Information, search and price dispersion. In: Hendershott, T. (Ed.), *Handbook on Economics and Information Systems*. North-Holland, Amsterdam, The Netherlands.

Baye, M., Gatti, J., Kattuman, P., Morgan, J., 2007. A Dashboard for Online Pricing. Working paper. University of California, Berkeley.

Bester, H., Petrakis, E., 1996. Coupons and oligopolistic price discrimination. International Journal of Industrial Organization 14, 227–242.

Brynjolfsson, E., Smith, M., 2000a. Frictionless commerce? A comparison of internet and conventional retailers. Management Science 46, 563–585.

Brynjolfsson, E., Smith, M., 2000b. The great equalizer? Consumer choice behavior and internet shopbots. Working paper. MIT.

Caminal, R., Matutes, C., 1990. Endogenous switching costs in a duopoly. International Journal of Industrial Organization 8, 353–373.

Chen, Yongmin, 1997. Paying customers to switch. Journal of Economics and Management Strategy 6, 877–897.

Chen, Yongmin, 2005. Oligopoly price discrimination by purchase history. *Pros and Cons of Price Discrimination*. Swedish Competition Authority, Stockholm.

Chen, Y., Zhang, Z., 2009. Dynamic targeted pricing with strategic consumers. International Journal of Industrial Organization 27, 43–50.

Chen, Yuxin, Narasimhan, C., Zhang, Z.J., 2001. Individual marketing with imperfect targetability. Marketing Science 20, 23–41.

Corts, K., 1998. Third-degree price discrimination in oligopoly: all-out competition and strategic commitment. RAND Journal of Economics 29, 306–323.

Esteves, R.B. 2004. Competitive Behaviour-Based Price Discrimination. PhD thesis, Oxford University.

Esteves, R.B., 2009a. Customer poaching and advertising. Journal of Industrial Economics 57, 112–146.

Esteves, R.B., 2009b. Price discrimination with partial information: does it pay-off? Economics Letters 105, 28–31.

Esteves, R.B., 2009c. A survey on the economics of behaviour-based price discrimination. Working paper, NIPE - WP 5/2009.

Fudenberg, D., Tirole, J., 2000. Customer poaching and brand switching. RAND Journal of Economics 31, 634–657.

Fudenberg, D., Villas-Boas, M., 2006. Behavior-based price discrimination and customer recognition. In: Hendershott, T. (Ed.), *Handbook on Economics and Information Systems*. North-Holland, Amsterdam, The Netherlands.

Narasimhan, C., 1988. Competitive promotional strategies. Journal of Business 61, 427–449.

Padilla, J., 1995. Revisiting dynamic duopoly with consumer switching costs. Journal of Economic Theory 67, 520–530.

Raju, J., Srinivasan, V., Rajiv, L., 1990. The effects of brand loyalty on competitive price promotional strategies. Management Science 36, 276–304.

Shaffer, G., Zhang, Z., 1995. Competitive coupon targeting. Marketing Science 14, 395–416.

Shaffer, G., Zhang, Z., 2000. Pay to switch or pay to stay: preference-based price discrimination in markets with switching costs. Journal of Economics and Management Strategy 9, 397–424.

Shilony, Y., 1977. Mixed pricing in oligopoly. Journal of Economic Theory 14, 373–388.

Stole, L., 2007. Price discrimination in competitive environments. In: Armstrong, M., Porter, R. (Eds.), The Handbook of Industrial Organization, Vol. 3. North-Holland, Amsterdam.

Taylor, C., 2003. Supplier surfing: competition and consumer behaviour in subscription markets. RAND Journal of Economics 34, 223–246.

Thisse, J., Vives, X., 1988. On the strategic choice of spatial price policy. American Economic Review 78, 122–137.

Varian, H., 1980. A model of sales. American Economic Review 70, 651–659.

Villas-Boas, M., 1999. Dynamic competition with customer recognition. RAND Journal of Economics 30, 604–631.

Zhang, J., 2008. The perils of customization: a model of endogenous segmentation and product design. Working paper. MIT Sloan School of Management.